# CHAPTER - 11
# DUAL NATURE OF RADIATION & MATTER - EXERCISE SOLUTIONS

**Question 11.1:**

Find the

**(a)** maximum frequency, and

**(b)** minimum wavelength of X-rays produced by 30 kV electrons.

**Answer 11.1:**

Potential of the electrons, V = 30 kV = $3 \times 10^4$ V

Hence, energy of the electrons, E = $3 \times 10^4$ eV

Where,

e = Charge on an electron = $1.6 \times 10^{-19}$ C

**(a)** Maximum frequency produced by the X-rays = v

The energy of the electrons is given by the relation:

E = hv

Where,

h = Planck's constant = $6.626 \times 10^{-34}$ Js

$$\therefore \nu = \frac{E}{h}$$

$$= \frac{1.6 \times 10^{-19} \times 3 \times 10^4}{6.626 \times 10^{-34}} = 7.24 \times 10^{18} \text{ Hz}$$

Hence, the maximum frequency of X-rays produced is $7.24 \times 10^{18}$ Hz.

**(b)** The minimum wavelength produced by the X-rays is given as:

$$\lambda = \frac{c}{\nu}$$

$$= \frac{3 \times 10^8}{7.24 \times 10^{18}} = 4.14 \times 10^{-11} \text{ m} = 0.0414 \text{ nm}$$

Hence, the minimum wavelength of X-rays produced is 0.0414 nm.

**Question 11.2:**

The work function of caesium metal is 2.14 eV. When light of frequency 6 $\times 10^{14}$ Hz is incident on the metal surface, photoemission of electrons occurs. What is the

**(a)** maximum kinetic energy of the emitted electrons,

**(b)** Stopping potential, and

**(c)** maximum speed of the emitted photoelectrons?

**Answer 11.2:**

Work function of caesium metal, $\phi_0 = 2.14 \text{ eV}$

Frequency of light, $\nu = 6.0 \times 10^{14} \text{ Hz}$

**(a)** The maximum kinetic energy is given by the photoelectric effect as:

$$K = h\nu - \phi_0$$

Where,

h = Planck's constant = $6.626 \times 10^{-34}$ Js

$$\therefore K = \frac{6.626 \times 10^{34} \times 6 \times 10^{14}}{1.6 \times 10^{-19}} - 2.14$$

$$= 2.485 - 2.140 = 0.345 \text{ eV}$$

Hence, the maximum kinetic energy of the emitted electrons is 0.345 eV.

**(b)** For stopping potential $V_0$, we can write the equation for kinetic energy as:

$$K = eV_0$$

$$\therefore V_0 = \frac{K}{e}$$

$$= \frac{0.345 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19}} = 0.345 \text{ V}$$

Hence, the stopping potential of the material is 0.345 V.

**(c)** Maximum speed of the emitted photoelectrons = v

Hence, the relation for kinetic energy can be written as:

$$K = \frac{1}{2}mv^2$$

Where,

m = Mass of an electron = $9.1 \times 10^{-31}$ kg

$$v^2 = \frac{2K}{m}$$

$$= \frac{2 \times 0.345 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}} = 0.1104 \times 10^{12}$$

$$\therefore v = 3.323 \times 10^5 \text{ m/s} = 332.3 \text{ km/s}$$

Hence, the maximum speed of the emitted photoelectrons is 332.3 km/s.

**Question 11.3:**

The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

**Answer 11.3:**

Photoelectric cut-off voltage, $V_0$ = 1.5 V

The maximum kinetic energy of the emitted photoelectrons is given as:

$$K_e = eV_0$$

Where,

e = Charge on an electron = $1.6 \times 10^{-19}$ C

$$\therefore K_e = 1.6 \times 10^{-19} \times 1.5$$
$$= 2.4 \times 10^{-19} \text{ J}$$

Therefore, the maximum kinetic energy of the photoelectrons emitted in the given experiment is $2.4 \times 10^{-19}$ J.

**Question 11.4:**

Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.

**(a)** Find the energy and momentum of each photon in the light beam,

**(b)** How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and

**(c)** How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

**Answer 11.4:**

Wavelength of the monochromatic light, $\lambda$ = 632.8 nm = $632.8 \times 10^{-9}$ m

Power emitted by the laser, P = 9.42 mW = $9.42 \times 10^{-3}$ W

Planck's constant, h = $6.626 \times 10^{-34}$ Js

Speed of light, c = $3 \times 10^{8}$ m/s

Mass of a hydrogen atom, m = $1.66 \times 10^{-27}$ kg

**(a)** The energy of each photon is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{632.8 \times 10^{-9}} = 3.141 \times 10^{-19} \text{ J}$$

The momentum of each photon is given as:

$$P = \frac{h}{\lambda}$$

$$= \frac{6.626 \times 10^{-34}}{632.8} = 1.047 \times 10^{-27} \text{ kg m s}^{-1}$$

**(b)** Number of photons arriving per second, at a target irradiated by the beam = n

Assume that the beam has a uniform cross-section that is less than the target area.

Hence, the equation for power can be written as:

$P = nE$

$\therefore n = \frac{P}{E}$

$= \frac{9.42 \times 10^{-3}}{3.141 \times 10^{-19}} \approx 3 \times 10^{16}$ photon/s

**(c)** Momentum of the hydrogen atom is the same as the momentum of the photon,

$p = 1.047 \times 10^{-27}$ kg m s$^{-1}$

Momentum is given as:

$p = mv$

Where,

v = Speed of the hydrogen atom

$\therefore v = \frac{p}{m}$

$= \frac{1.047 \times 10^{-27}}{1.66 \times 10^{-27}} = 0.621$ m/s

**Question 11.5:**

The energy flux of sunlight reaching the surface of the earth is $1.388 \times 10^3$ W/m$^2$. How many photons (nearly) per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.

**Answer 11.5:**

Energy flux of sunlight reaching the surface of earth, $\Phi = 1.388 \times 10^3$ W/m$^2$

Hence, power of sunlight per square metre, $P = 1.388 \times 10^3$ W

Speed of light, $c = 3 \times 10^8$ m/s

Planck's constant, $h = 6.626 \times 10^{-34}$ Js

Average wavelength of photons present in sunlight, $\lambda = 550$ nm

$= 550 \times 10^{-9}$ m

Number of photons per square metre incident on earth per second = n Hence, the equation for power can be written as:

$$P = nE$$

$$\therefore n = \frac{P}{E} = \frac{P\lambda}{hc}$$

$$= \frac{1.388 \times 10^{3} \times 550 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^{8}} = 3.84 \times 10^{21} \text{ photons/m}^2\text{/s}$$

Therefore, every second, $3.84 \times 10^{21}$ photons are incident per square metre on earth.

**Question 11.6:**

In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be $4.12 \times 10^{-15}$ V s. Calculate the value of Planck's constant.

**Answer 11.6:**

The slope of the cut-off voltage (V) versus frequency (ν) of an incident light is given as:

$$\frac{V}{\nu} = 4.12 \times 10^{-15} \text{ Vs}$$

$V$ is related to frequency by the equation:

$$h\nu = eV$$

Where,

e = Charge on an electron = $1.6 \times 10^{-19}$ C

h = Planck's constant

$$\therefore h = e \times \frac{V}{\nu}$$

$$= 1.6 \times 10^{-19} \times 4.12 \times 10^{-15} = 6.592 \times 10^{-34} \text{ Js}$$

Therefore, the value of Planck's constant is $6.592 \times 10^{-34}$ Js.

**Question 11.7:**

A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm. **(a)** What is the energy per photon associated with the sodium light? **(b)** At what rate are the photons delivered to the sphere?

**Answer 11.7:**

Power of the sodium lamp, P = 100 W

Wavelength of the emitted sodium light, $\lambda$ = 589 nm = $589 \times 10^{-9}$ m

Planck's constant, h = $6.626 \times 10^{-34}$ Js

Speed of light, c = $3 \times 10^{8}$ m/s

**(a)** The energy per photon associated with the sodium light is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{589 \times 10^{-9}} = 3.37 \times 10^{-19} \text{ J}$$

$$= \frac{3.37 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.11 \text{ eV}$$

**(b)** Number of photons delivered to the sphere = n.

The equation for power can be written as:

$$P = nE$$

$$\therefore n = \frac{P}{E}$$

$$= \frac{100}{3.37 \times 10^{-19}} = 2.96 \times 10^{20} \text{ photons/s}$$

Therefore, every second, $2.96 \times 10^{20}$ photons are delivered to the sphere.

**Question 11.8:**

The threshold frequency for a certain metal is $3.3 \times 10^{14}$ Hz. If light of frequency $8.2\times10^{14}$ Hz is incident on the metal, predict the cut-off voltage for the photoelectric emission.

**Answer 11.8:**

Threshold frequency of the metal, $\nu_0 = 3.3 \times 10^{14}$ Hz

Frequency of light incident on the metal, $\nu = 8.2 \times 10^{14}$ Hz

Charge on an electron, e = $1.6 \times 10^{-19}$ C

Planck's constant, h = $6.626 \times 10^{-34}$ Js

Cut-off voltage for the photoelectric emission from the metal = $V_0$

The equation for the cut-off energy is given as:

$$eV_0 = h(\nu - \nu_0)$$

$$V_0 = \frac{h(\nu - \nu_0)}{e}$$

$$= \frac{6.626 \times 10^{-34} \times (8.2 \times 10^{14} - 3.3 \times 10^{14})}{1.6 \times 10^{-19}} = 2.0292 \text{ V}$$

Therefore, the cut-off voltage for the photoelectric emission is 2.0292 V.

**Question 11.9:**

The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

**Answer 11.9:**

No

Work function of the metal, $\phi_0 = 4.2$ eV

Charge on an electron, e = $1.6 \times 10^{-19}$ C

Planck's constant, h = $6.626 \times 10^{-34}$ Js

Wavelength of the incident radiation, $\lambda$ = 330 nm = $330 \times 10^{-9}$ m

Speed of light, c = $3 \times 10^{8}$ m/s

The energy of the incident photon is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{330 \times 10^{-9}} = 6.0 \times 10^{-19} \text{ J}$$

$$= \frac{6.0 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.76 \text{ eV}$$

It can be observed that the energy of the incident radiation is less than the work function of the metal. Hence, no photoelectric emission will take place.

**Question 11.10:**

Light of frequency $7.21 \times 10^{14}$ Hz is incident on a metal surface. Electrons with a maximum speed of $6.0 \times 10^{5}$ m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

**Answer 11.10:**

Frequency of the incident photon, $\nu = 488 \text{ nm} = 488 \times 10^{-9} \text{ m}$

Maximum speed of the electrons, v = $6.0 \times 10^{5}$ m/s

Planck's constant, h = $6.626 \times 10^{-34}$ Js

Mass of an electron, m = $9.1 \times 10^{-31}$ kg

For threshold frequency $\nu_0$, the relation for kinetic energy is written as:

$$\frac{1}{2}mv^2 = h(\nu - \nu_0)$$

$$\nu_0 = \nu - \frac{mv^2}{2h}$$

$$= 7.21 \times 10^{14} - \frac{(9.1 \times 10^{-31}) \times (6 \times 10^{5})^2}{2 \times (6.626 \times 10^{-34})}$$

$$= 7.21 \times 10^{14} - 2.472 \times 10^{14}$$

$$= 4.738 \times 10^{14} \text{ Hz}$$

Therefore, the threshold frequency for the photoemission of electrons is $4.738 \times 10^{14}$ Hz.

**Question 11.11:**

Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

**Answer 11.11:**

Wavelength of light produced by the argon laser, $\lambda = 488$ nm $= 488 \times 10^{-9}$ m

Stopping potential of the photoelectrons, $V_0 = 0.38$ V

1eV $= 1.6 \times 10^{-19}$ J

$\therefore V_0 = \frac{0.38}{1.6 \times 10^{-19}}$ eV

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Speed of light, $c = 3 \times 10$ m/s

From Einstein's photoelectric effect, we have the relation involving the work function $\Phi_0$ of the material of the emitter as:

$$eV_0 = \frac{hc}{\lambda} - \phi_0$$

$$\phi_0 = \frac{hc}{\lambda} - eV_0$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 488 \times 10^{-9}} - \frac{1.6 \times 10^{-19} \times 0.38}{1.6 \times 10^{-19}}$$

$$= 2.54 - 0.38 = 2.16 \text{ eV}$$

Therefore, the material with which the emitter is made has the work function of 2.16 eV.

**Question 11.12:**

Calculate the

**(a)** momentum, and

**(b)** de Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

**Answer 11.2:**

Potential difference, V = 56 V

Planck's constant, h = $6.6 \times 10^{-34}$ Js

Mass of an electron, m = $9.1 \times 10^{-31}$ kg

Charge on an electron, e = $1.6 \times 10^{-19}$ C

**(a)** At equilibrium, the kinetic energy of each electron is equal to the accelerating potential, i.e., we can write the relation for velocity (v) of each electron as:

$$\frac{1}{2}mv^2 = eV$$

$$v^2 = \frac{2eV}{m}$$

$$\therefore v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 56}{9.1 \times 10^{-31}}}$$

$$= \sqrt{19.69 \times 10^{12}} = 4.44 \times 10^6 \text{ m/s}$$

The momentum of each accelerated electron is given as:

p = mv

= $9.1 \times 10^{-31} \times 4.44 \times 10^6$

= $4.04 \times 10^{-24}$ kg m $s^{-1}$

Therefore, the momentum of each electron is $4.04 \times 10^{-24}$ kg m $s^{-1}$.

**(b)** De Broglie wavelength of an electron accelerating through a potential V, is given by the relation:

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ Å}$$

$$= \frac{12.27}{\sqrt{56}} \times 10^{-10} \text{ m}$$

$$= 0.1639 \text{ nm}$$

Therefore, the de Broglie wavelength of each electron is 0.1639 nm.

**Question 11.13:**

What is the

**(a)** momentum,

**(b)** speed, and

**(c)** de Broglie wavelength of an electron with kinetic energy of 120 eV.

**Answer 11.13:**

Kinetic energy of the electron, $E_k$ = 120 eV

Planck's constant, h = $6.6 \times 10^{-34}$ Js

Mass of an electron, m = $9.1 \times 10^{-31}$ kg

Charge on an electron, e = $1.6 \times 10^{-19}$ C

**(a)** For the electron, we can write the relation for kinetic energy as:

$$E_k = \frac{1}{2}mv^2$$

Where, v = Speed of the electron

$$\therefore v^2 = \sqrt{\frac{2eE_k}{m}}$$

$$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 120}{9.1 \times 10^{-31}}}$$

$$= \sqrt{42.198 \times 10^{12}} = 6.496 \times 10^6 \text{ m/s}$$

Momentum of the electron, p = mv = $9.1 \times 10^{-31} \times 6.496 \times 10^6$

= $5.91 \times 10^{-24}$ kg $ms^{-1}$

Therefore, the momentum of the electron is $5.91 \times 10^{-24}$ kg m $s^{-1}$.

**(b)** Speed of the electron, $v = 6.496 \times 10^{6}$ m/s

**(c)** De Broglie wavelength of an electron having a momentum p, is given as:

$$\lambda = \frac{h}{p}$$

$$= \frac{6.6 \times 10^{-34}}{5.91 \times 10^{-24}} = 1.116 \times 10^{-10} \text{ m}$$

$$= 0.112 \text{ nm}$$

Therefore, the de Broglie wavelength of the electron is 0.112 nm.

**Question 11.14:**

The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which

**(a)** an electron, and

**(b)** a neutron, would have the same de Broglie wavelength.

**Answer 11.14:**

Wavelength of light of a sodium line, $\lambda = 589$ nm $= 589 \times 10^{-9}$ m

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Mass of a neutron, $m_n = 1.66 \times 10^{-27}$ kg

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

**(a)** For the kinetic energy K, of an electron accelerating with a velocity v, we have the relation:

$$K = \frac{1}{2} m_e v^2 \quad \ldots (1)$$

We have the relation for de Broglie wavelength as:

$$\lambda = \frac{h}{m_e v}$$

$$\therefore v^2 = \frac{h^2}{\lambda^2 m_e^2} \qquad \dots(2)$$

Substituting equation (2) in equation (1), we get the relation:

$$K = \frac{1}{2}\frac{m_e h^2}{\lambda^2 m_e^2} = \frac{h^2}{2\lambda^2 m_e} \qquad \dots(3)$$

$$= \frac{\left(6.6 \times 10^{-34}\right)^2}{2 \times \left(589 \times 10^{-9}\right)^2 \times 9.1 \times 10^{-31}}$$

$$\approx 6.9 \times 10^{-25} \text{ J}$$

$$= \frac{6.9 \times 10^{-25}}{1.6 \times 10^{-19}} = 4.31 \times 10^{-6} \text{ eV} = 4.31 \mu\text{eV}$$

Hence, the kinetic energy of the electron is $6.9 \times 10^{-25}$ J or 4.31 μeV.

**(b)** Using equation (3), we can write the relation for the kinetic energy of the neutron as:

$$\frac{h^2}{2\lambda^2 m_n}$$

$$= \frac{\left(6.6 \times 10^{-34}\right)^2}{2 \times \left(589 \times 10^{-9}\right)^2 \times 1.66 \times 10^{-27}}$$

$$= 3.78 \times 10^{-28} \text{ J}$$

$$= \frac{3.78 \times 10^{-28}}{1.6 \times 10^{-19}} = 2.36 \times 10^{-9} \text{ eV} = 2.36 \text{ neV}$$

Hence, the kinetic energy of the neutron is $3.78 \times 10^{-28}$ J or 2.36 neV.

**Question 11.15:**

What is the de Broglie wavelength of

**(a)** a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s,

**(b)** a ball of mass 0.060 kg moving at a speed of 1.0 m/s, and

**(c)** a dust particle of mass $1.0 \times 10^{-9}$ kg drifting with a speed of 2.2 m/s?

**Answer 11.15:**

**(a)** Mass of the bullet, m = 0.040 kg

Speed of the bullet, v = 1.0 km/s = 1000 m/s

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

De Broglie wavelength of the bullet is given by the relation:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{0.040 \times 1000} = 1.65 \times 10^{-35} \text{ m}$$

**(b)** Mass of the ball, m = 0.060 kg

Speed of the ball, v = 1.0 m/s

De Broglie wavelength of the ball is given by the relation:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{0.060 \times 1} = 1.1 \times 10^{-32} \text{ m}$$

**(c)** Mass of the dust particle, $m = 1 \times 10^{-9}$ kg

Speed of the dust particle, v = 2.2 m/s

De Broglie wavelength of the dust particle is given by the relation:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{2.2 \times 1 \times 10^{-9}} = 3.0 \times 10^{-25} \text{ m}$$

**Question 11.16:**

An electron and a photon each have a wavelength of 1.00 nm. Find

**(a)** their momenta,

**(b)** the energy of the photon, and

**(c)** the kinetic energy of electron.

**Answer 11.16:**

Wavelength of an electron $(\lambda_e)$ and a photon $(\lambda_p)$, $\lambda_e = \lambda_p = \lambda = 1$ nm

$= 1 \times 10^{-9}$ m

Planck's constant, h = $6.63 \times 10^{-34}$ Js

**(a)** The momentum of an elementary particle is given by de Broglie relation:

$$\lambda = \frac{h}{p}$$

$$p = \frac{h}{\lambda}$$

It is clear that momentum depends only on the wavelength of the particle. Since the wavelengths of an electron and a photon are equal, both have an equal momentum.

$$\therefore p = \frac{6.63 \times 10^{-34}}{1 \times 10^{-9}} = 6.63 \times 10^{-25} \text{ kg m s}^{-1}$$

**(b)** The energy of a photon is given by the relation:

$$E = \frac{hc}{\lambda}$$

Where,

Speed of light, c = $3 \times 10^{8}$ m/s

$$\therefore E = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1 \times 10^{-9} \times 1.6 \times 10^{-19}}$$

$$= 1243.1 \text{ eV} = 1.243 \text{ keV}$$

Therefore, the energy of the photon is 1.243 keV.

**(c)** The kinetic energy (K) of an electron having momentum p, is given by the relation:

$$K = \frac{1}{2}\frac{p^2}{m}$$

Where,

m = Mass of the electron = $9.1 \times 10^{-31}$ kg

p = $6.63 \times 10^{-25}$ kg m $s^{-1}$

$$\therefore K = \frac{1}{2} \times \frac{\left(6.63 \times 10^{-25}\right)^2}{9.1 \times 10^{-31}} = 2.415 \times 10^{-19} \text{ J}$$

$$= \frac{2.415 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.51 \text{ eV}$$

Hence, the kinetic energy of the electron is 1.51 eV.

**Question 11.17:**

**(a)** For what kinetic energy of a neutron will the associated de Broglie wavelength be $1.40 \times 10^{-10}$ m?

**(b)** Also find the de Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of (3/2) kT at 300 K.

**Answer 11.17:**

**(a)** De Broglie wavelength of the neutron, $\lambda = 1.40 \times 10^{-10}$ m

Mass of a neutron, $m_n = 1.66 \times 10^{-27}$ kg

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

Kinetic energy (K) and velocity (v) are related as:

$$K = \frac{1}{2} m_n v^2 \quad \ldots (1)$$

De Broglie wavelength (λ) and velocity (v) are related as:

$$\lambda = \frac{h}{m_n v} \quad \ldots (2)$$

Using equation (2) in equation (1), we get:

$$K = \frac{1}{2}\frac{m_n h^2}{\lambda^2 m_n^2} = \frac{h^2}{2\lambda^2 m_n}$$

$$= \frac{\left(6.63 \times 10^{-34}\right)^2}{2 \times \left(1.40 \times 10^{-10}\right)^2 \times 1.66 \times 10^{-27}} = 6.75 \times 10^{-21} \text{ J}$$

$$= \frac{6.75 \times 10^{-21}}{1.6 \times 10^{-19}} = 4.219 \times 10^{-2} \text{ eV}$$

Hence, the kinetic energy of the neutron is $6.75 \times 10^{-21}$ J or $4.219 \times 10^{-2}$ eV.

**(b)** Temperature of the neutron, T = 300 K

Boltzmann constant, k = $1.38 \times 10^{-23}$ kg $m^2$ $s^{-2}$ $K^{-1}$

Average kinetic energy of the neutron:

$$K' = \frac{3}{2}kT$$

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.21 \times 10^{-21} \text{ J}$$

The relation for the de Broglie wavelength is given as:

$$\lambda' = \frac{h}{\sqrt{2K' m_n}}$$

Where,

$m_n = 1.66 \times 10^{-27}$ kg

$h = 6.6 \times 10^{-34}$ Js

$K' = 6.75 \times 10^{-21}$ J

$$\therefore \lambda' = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.21 \times 10^{-21} \times 1.66 \times 10^{-27}}} = 1.46 \times 10^{-10} \text{ m} = 0.146 \text{ nm}$$

Therefore, the de Broglie wavelength of the neutron is 0.146 nm.

**Question 11.18:**

Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

**Answer 11.18:**

The momentum of a photon having energy (hv) is given as:

$$p = \frac{h\nu}{c} = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{p} \qquad \text{... } (i)$$

Where,

λ = Wavelength of the electromagnetic radiation

c = Speed of light

h = Planck's constant

De Broglie wavelength of the photon is given as:

$$\lambda = \frac{h}{mv}$$

$$\text{But } p = mv$$

$$\therefore \lambda = \frac{h}{p} \qquad \text{... } (ii)$$

Where, m = Mass of the photon

v = Velocity of the photon

Hence, it can be inferred from equations (i) and (ii) that the wavelength of the electromagnetic radiation is equal to the de Broglie wavelength of the photon.

What is the de Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root-mean square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u)

**Answer 11.19:**

Temperature of the nitrogen molecule, T = 300 K

Atomic mass of nitrogen = 14.0076 u

Hence, mass of the nitrogen molecule, m = 2 × 14.0076 = 28.0152 u

But 1 u = $1.66 \times 10^{-27}$ kg

∴m = 28.0152 ×$1.66 \times 10^{-27}$ kg

Planck's constant, h = $6.63 \times 10^{-34}$ Js

Boltzmann constant, k = $1.38 \times 10^{-23}$ J $K^{-1}$

We have the expression that relates mean kinetic energy $\left(\frac{3}{2}kT\right)$ of the nitrogen molecule with the root mean square speed $(v_{rms})$ as:

$$\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$$

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

Hence, the de Broglie wavelength of the nitrogen molecule is given as:

$$\lambda = \frac{h}{mv_{rms}} = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 28.0152 \times 1.66 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}}$$

$$= 0.028 \times 10^{-9} \text{ m}$$
$$= 0.028 \text{ nm}$$

Therefore, the de Broglie wavelength of the nitrogen molecule is 0.028 nm.