

CHAPTER - 1

NUMBER SYSTEMS

EXERCISE 1.1

Q.1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Sol. Yes, zero is a rational number. It can be written as $\frac{0}{1}, \frac{0}{2}$, etc., in the form

$\frac{p}{q}$, where p and q are integers and $q \neq 0$. **Ans.**

Q.2. Find six rational numbers between 3 and 4.

Sol. To find six rational numbers between 3 and 4 denominator should be made equal to $6 + 1 = 7$.

$$\text{Therefore, } 3 = \frac{3 \times 7}{7} = \frac{21}{7} \quad 4 = \frac{4 \times 7}{7} = \frac{28}{7}$$

Six rational numbers between 3 and 4 can be found by varying the numerator between 21 and 28.

Or, the numbers are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$. **Ans.**

Q.3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Sol. To find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$, we may add the given numbers and divide by 2, and repeat the process.

$$\frac{\frac{3}{5} + \frac{4}{5}}{2} = \frac{7}{5 \times 2} = \frac{7}{10} = x_1$$

$$\frac{\frac{7}{10} + \frac{4}{5}}{2} = \frac{7+8}{10} = \frac{15}{10}$$

$$\text{Next rational number} = \frac{15}{10 \times 2} = \frac{15}{20} = \frac{3}{4} = x_2$$

$$\frac{\frac{3}{4} + \frac{4}{5}}{2} = \frac{15+16}{20} = \frac{31}{20}$$

$$\text{Next rational number} = \frac{31}{20 \times 2} = \frac{31}{40} = x_3$$

$$\frac{\frac{31}{40} + \frac{4}{5}}{2} = \frac{31+32}{40} = \frac{63}{40}$$

$$\text{Next rational number} = \frac{63}{40 \times 2} = \frac{63}{80} = x_4$$

$$\frac{\frac{63}{80} + \frac{4}{5}}{2} = \frac{63+64}{80} = \frac{127}{80}$$

$$\text{Next rational number} = \frac{127}{80 \times 2} = \frac{127}{160} = x_5$$

$$x_1 = \frac{7}{10}, x_2 = \frac{3}{4}, x_3 = \frac{31}{40}, x_4 = \frac{63}{80}, x_5 = \frac{127}{160}. \quad \text{Ans.}$$

(**Note :** Many answers are possible. There are of course infinitely many rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.)

Q.4. *State whether the following statements are true or false. Give reasons for your answers.*

- (i) *Every natural number is a whole number.*
- (ii) *Every integer is a whole number.*
- (iii) *Every rational number is a whole number.*

Sol.

- (i) True, since the collection of whole numbers contains all the natural numbers and in addition zero.
- (ii) False. Negative integers are not whole numbers.
- (iii) False. Numbers such as $\frac{2}{3}, \frac{3}{4}, \frac{-3}{5}$, etc., are rational numbers but not whole numbers.

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EXERCISE 1.2

Q.1. *State whether the following statements are true or false. Justify your answers.*

- (i) *Every irrational number is a real number.*
- (ii) *Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.*
- (iii) *Every real number is an irrational number.*

Sol. (i) True. All irrational and rational numbers together make up the collection of real numbers $\mathbb{R}$.

(ii) False, e.g. between $\sqrt{2}$ and $\sqrt{3}$ there are infinitely many numbers and these can not be represented in the form $\sqrt{m}$, where m is a natural number.

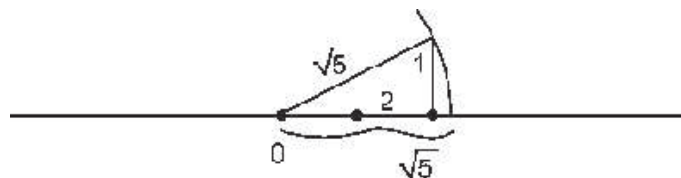
(iii) False. All rational numbers are also real numbers.

Q.2. *Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.*

Sol. Square roots of positive integers are not irrational. For example, $\sqrt{4} = 2$, which is a rational number.

Q.3. *Show how $\sqrt{5}$ can be represented on the number line.*

Sol. To represent $\sqrt{5}$ on the number line we take a length of two units from 0 on the number line in positive direction and one unit perpendicular to it. The hypotenuse of the triangle thus formed is of length $\sqrt{5}$. Then with the help of a divider a length equal to the hypotenuse of $\sqrt{5}$ units can be cut on the number line.



EXERCISE 1.3

Q.1. Write the following in decimal form and say what kind of decimal expansion each has :

$$(i) \frac{36}{100} \quad (ii) \frac{1}{11} \quad (iii) 4\frac{1}{8} \quad (iv) \frac{3}{13} \quad (v) \frac{2}{11} \quad (vi) \frac{329}{400}$$

Sol. (i) 0.36, terminating. (ii) $0.\overline{09}$, recurring non-terminating.
 (iii) 4.125, terminating. (iv) $0.\overline{230769}$, recurring non-terminating.
 (v) $0.\overline{18}$, non-terminating recurring. (vi) 0.8225, terminating.

Q.2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

Sol. $\frac{2}{7} = 2 \times \frac{1}{7} = 0.\overline{285714}$, $\frac{3}{7} = 3 \times \frac{1}{7} = 0.\overline{428571}$ $\frac{4}{7} = 4 \times \frac{1}{7} = 0.\overline{571428}$,
 $\frac{5}{7} = 5 \times \frac{1}{7} = 0.\overline{714285}$ $\frac{6}{7} = 6 \times \frac{1}{7} = 0.\overline{857142}$

Q.3. Express the following in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

$$(i) 0.\overline{6} \quad (ii) 0.4\overline{7} \quad (iii) 0.\overline{001}$$

Sol. (i) $x = 0.\overline{6} = 0.666 \dots$ to be written in $\frac{p}{q}$ form.

One digit 6 is repeating.

We multiply it with 10 on both sides.

$$\begin{aligned} 10x &= 6.\overline{6} \\ \Rightarrow 10x &= 6 + x \\ \Rightarrow 10x - x &= 6 \\ 9x &= 6 \\ \Rightarrow x &= \frac{6}{9} = \frac{2}{3} \quad \text{Ans.} \end{aligned}$$

(ii) $x = 0.4\overline{7} = 0.4777 \dots$

One digit is repeating.

We multiply by 10 on both sides.

$$\therefore 10x = 4.\overline{7}$$

$$= 4.3 + .4\overline{7}$$

$$= 4.3 + x$$

$$\Rightarrow 9x = 4.3$$

$$\Rightarrow x = \frac{4.3}{9} = \frac{43}{90} \quad \text{Ans.}$$

(iii) $x = 0.\overline{001}$.

Here three digits repeats; we multiply with 1000.

$$\therefore 1000x = 1.\overline{001}$$

$$1000x = 1 + x$$

$$\Rightarrow 1000x - x = 1$$

$$\Rightarrow 999x = 1$$

$$\Rightarrow x = \frac{1}{999} \quad \text{Ans.}$$

Q.4. Express $0.99999 \dots$ in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Sol. $x = 0.99999 \dots = 0.\overline{9}$

One digit repeat; we multiply by 10.

$$10x = 9.\overline{9}$$

$$\Rightarrow 10x = 9 + x$$

$$\Rightarrow 9x = 9$$

$$x = 1 \quad \text{Ans.}$$

The answer makes sense as $0.\overline{9}$ is infinitely close to 1, i.e., we can make the difference between 1 and $0.99 \dots$ as small as we wish by taking enough 9's.

Q.5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Sol. The maximum number of digits in the repeating block is 16 (< 17).

Division gives $\frac{1}{17} = 0.\overline{0588235294117647}$

The repeating block has 16 digits. **Ans.**

Q.6. Look at several examples of rational numbers in the form $\frac{p}{q}$ ($q \neq 0$), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Sol. $\frac{2}{5} = 0.4$, $\frac{3}{2} = 1.5$, $\frac{7}{8} = 0.875$, $\frac{7}{10} = 0.7$.

All the denominators are either 2 (or its power), 5 (or its power) or a combination of both. **Ans.**

Q.7. Write three numbers whose decimal expansion are non-terminating non-recurring.

Sol. 7.314114111411114.....

0.101002000300004.....

$\pi = 3.1416$ **Ans.**

Q.8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Sol. $\frac{5}{7} = 0.\overline{714285}$ $\frac{9}{11} = 0.\overline{81}$

There are an infinite number of irrational numbers between these two numbers. We may choose any three of them, e.g.

0.7234596.....

0.7425735.....

0.78123957..... **Ans.**

Q.9. Classify the following numbers as rational or irrational :

(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796

(iv) 7.478478 (v) 1.101001000100001....

Sol. Rational — $\sqrt{225} = 15$, and 0.3796

Irrational — $\sqrt{23}$, 7.478478....., 1.101001000100001.... **Ans.**

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EXERCISE 1.4

Q.1. Visualise 3.765 on the number line, using successive magnification.

Sol. Step one — The given number lies between 3 and 4.

Step two — Magnify the interval between 3 and 4 and divide it into 10 equal parts.

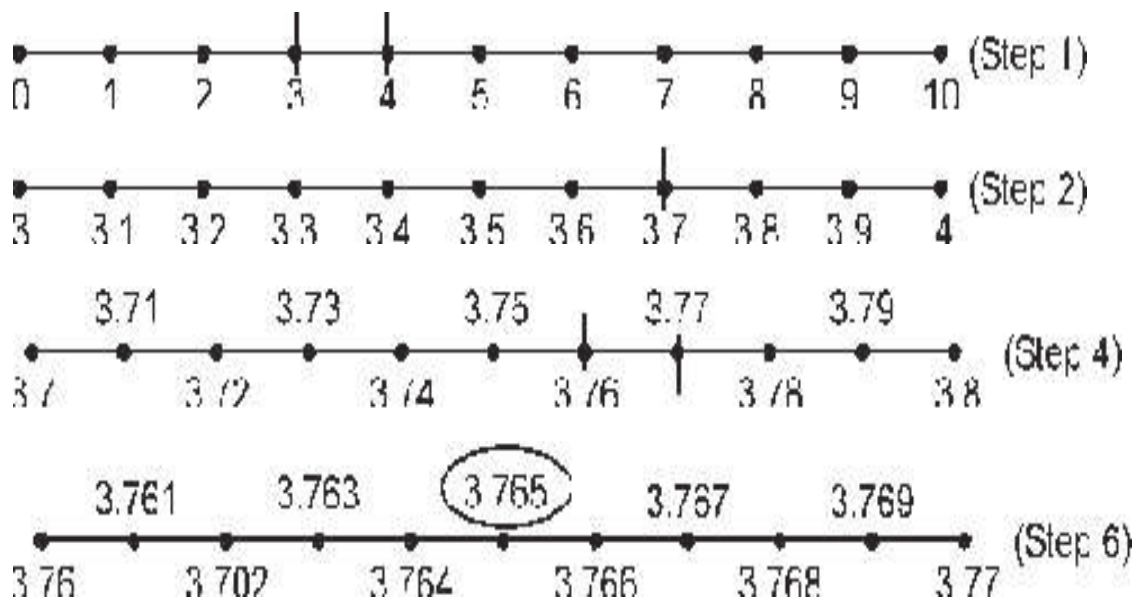
Step three — The given number lies between 3.7 and 3.8.

Step four — Divide the interval between 3.7 and 3.8 into ten equal parts and magnify it.

Step five — The given number lies between 3.76 and 3.77.

Step six — Magnify the interval between 3.76 and 3.77 and divide it into ten equal parts.

Step seven — 3.765 is the fifth division in this magnification.



Q.2. Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Sol. Step one — On the number line the given number $4.\overline{26}$ lies between 4 and 5. (For four decimal places number is 4.2626.)

Step two — Magnify the interval between 4 and 5 and divide it into 10 equal parts.

Step three — The given number 4.2626 lies between 4.2 and 4.3.

Step four — Magnify the interval between 4.2 and 4.3 and divide it into ten equal parts.

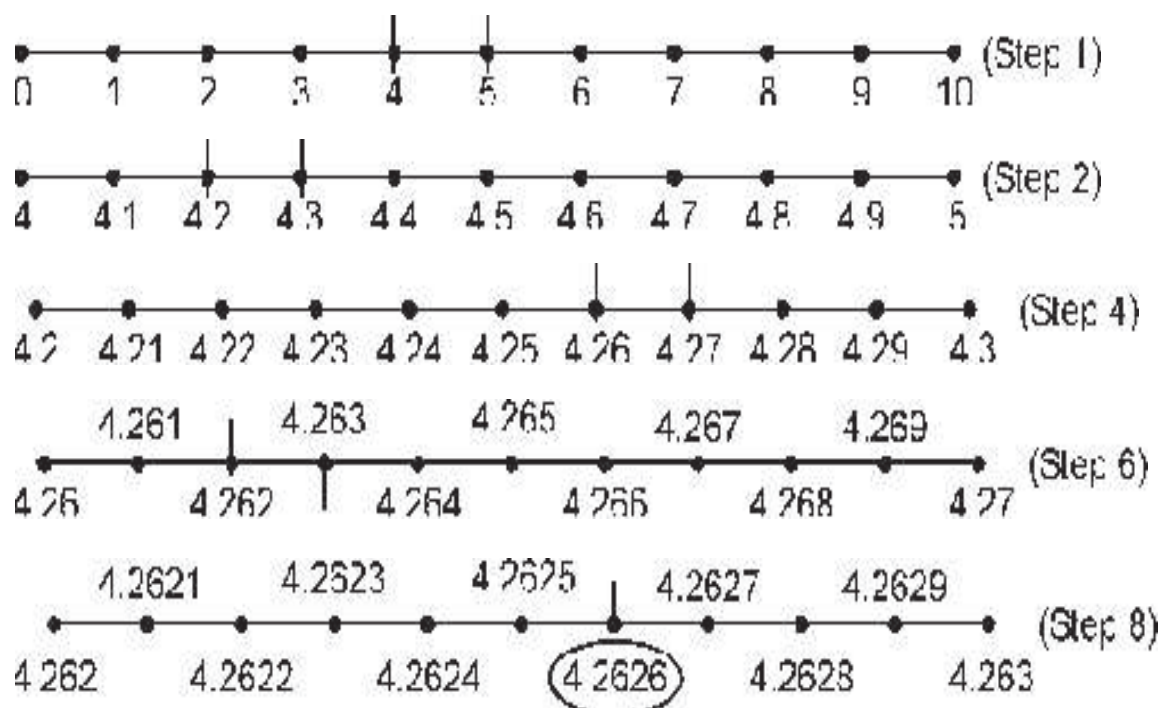
Step five — The given number falls between 4.26 and 4.27.

Step six — Magnify the interval between 4.26 and 4.27 and divide it into ten equal parts.

Step seven — The given number lies between 4.262 and 4.263.

Step eight — Magnify the interval between 4.262 and 4.263 and divide it into ten equal parts.

Step nine — The given number is the sixth division of the given interval.



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EXERCISE 1.5

Q.1. Classify the following numbers as rational or irrational :

$$(i) 2 - \sqrt{5} \quad (ii) (3 + \sqrt{23}) - \sqrt{23} \quad (iii) \frac{2\sqrt{7}}{7\sqrt{7}} \quad (iv) \frac{1}{\sqrt{2}} \quad (v) 2\pi$$

Ans. (i) $2 - \sqrt{5}$, (iv) $\frac{1}{\sqrt{2}}$ and, (v) 2π are irrational

(ii) $(3 + \sqrt{23}) - \sqrt{23} = 3$, and (iii) $\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$ are rational **Ans.**

Q.2. Simplify each of the following expressions :

$$(i) (3 + \sqrt{3})(2 + \sqrt{2}) \quad (ii) (3 + \sqrt{3})(3 - \sqrt{3})$$

$$(iii) (\sqrt{5} + \sqrt{2})^2 \quad (iv) (\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$$

Ans. (i) $(3 + \sqrt{3})(2 + \sqrt{2}) = 3 \times 2 + 3 \times \sqrt{2} + \sqrt{3} \times 2 + \sqrt{3} \times \sqrt{2}$
 $= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$ **Ans.**

(ii) $(3 + \sqrt{3})(3 - \sqrt{3})$

Using identity $(a + b)(a - b) = a^2 - b^2$, it equals, $3^2 - 3 = 9 - 3 = 6$. **Ans.**

(iii) $(\sqrt{5} + \sqrt{2})^2$

Using identity $(a + b)^2 = a^2 + b^2 + 2ab$, we have

$$(\sqrt{5} + \sqrt{2})^2 = 5 + 2 + 2\sqrt{2}\sqrt{5} = 7 + 2\sqrt{10}$$
 Ans.

(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

Using identity $(a + b)(a - b) = a^2 - b^2$ we have

$$(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = (5 - 2) = 3$$
 Ans.

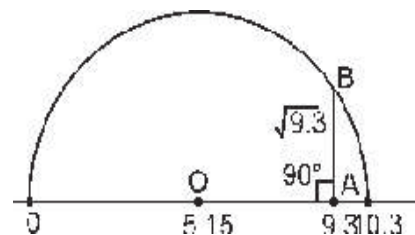
Q.3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Ans. With a scale or tape we get only an approximate rational number as the result of our measurement. That is why π can be approximately represented as a quotient of two rational numbers. As a matter of mathematical truth it is irrational.

Q.4. Represent $\sqrt{9.3}$ on the number line.

Sol. To represent $\sqrt{9.3}$, draw a segment of 9.3 units on the number line. Let A represent 9.3

Extend it by 1 cm. Show point $\frac{10.3}{2}$



= 5.15 by on the number line. With 'O' as centre and radius 5.15 units, draw a semicircle. Draw AB perpendicular to OA to cut the hemisphere at B. The length AB is $\sqrt{9.3}$ units.

Q.5. Rationalise the denominators of the following :

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$ (iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Sol. (i) $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$ **Ans.**

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$ **Ans.**

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$ **Ans.**

(iv) $\frac{1}{\sqrt{7}-2} = \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$ **Ans.**

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NUMBER SYSTEMS

EXERCISE 1.6

Q.1. Find : (i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

Sol. (i) $64^{\frac{1}{2}} = (8^2)^{\frac{1}{2}} = 8$ **Ans.** (ii) $32^{\frac{1}{5}} = (2 \times 2 \times 2 \times 2 \times 2)^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} = 2$ **Ans.**
 (iii) $125^{\frac{1}{3}} = (5 \times 5 \times 5)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5$ **Ans.**

Q.2. Find : (i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$ (iv) $125^{\frac{-1}{3}}$

Sol. (i) $9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3 = (3)^3 = 27$ **Ans.** (ii) $32^{\frac{2}{5}} = \left(32^{\frac{1}{5}}\right)^2 = (2)^2 = 4$ **Ans.**
 (iii) $16^{\frac{3}{4}} = \left(16^{\frac{1}{4}}\right)^3 = (2)^3 = 8$ **Ans.** (iv) $125^{\frac{-1}{3}} = \frac{1}{(125)^{\frac{1}{3}}} = \frac{1}{5}$ **Ans.**

Q.3. Simplify : (i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ (iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

Sol. (i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = 2^{\left(\frac{2}{3} + \frac{1}{5}\right)} = 2^{\frac{13}{15}}$ **Ans.** (ii) $\left(\frac{1}{3^3}\right)^7 = (3^{-3})^7 = (3)^{-21}$ **Ans.**

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2} - \frac{1}{4}} = 11^{\frac{1}{4}}$ **Ans.** (iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (56)^{\frac{1}{2}}$ **Ans.**