

## CHAPTER - 2

### POLYNOMIALS

#### EXERCISE 2.1

**Q.1.** Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i)  $4x^2 - 3x + 7$  (ii)  $y^2 + \sqrt{2}$  (iii)  $3\sqrt{t} + t\sqrt{2}$  (iv)  $y + \frac{2}{y}$  (v)  $x^{10} + y^3 + t^{50}$

**Sol.** (i) Polynomial in one variable,  $x$  **Ans.**

(ii) Polynomial in one variable,  $y$ . **Ans.**

(iii)  $3\sqrt{t} + t\sqrt{2}$  is not a polynomial as power of  $t$  in  $\sqrt{t}$  is not a whole number. **Ans.**

(iv)  $y + \frac{2}{y}$  is not a polynomial as power of  $y$  in second term, i.e.,  $\frac{1}{y} = y^{-1}$  is not a whole number. **Ans.**

(v)  $x^{10} + y^3 + t^{50}$  is not a polynomial in one variable but a polynomial in three variables  $x$ ,  $y$  and  $t$ . **Ans.**

**Q.2.** Write the coefficients of  $x^2$  in each of the following :

(i)  $2 + x^2 + x$  (ii)  $2 - x^2 + x^3$  (iii)  $\frac{\pi}{2}x^2 + x$  (iv)  $\sqrt{2}x - 1$

**Sol.** (i) In  $2 + x^2 + x$ , coefficient of  $x^2$  is 1. **Ans.**

(ii) In  $2 - x^2 + x^3$ , coefficient of  $x^2$  is  $-1$ . **Ans.**

(iii)  $\frac{\pi}{2}x^2 + x$ , coefficient of  $x^2$  is  $\frac{\pi}{2}$ . **Ans.**

(iv)  $\sqrt{2}x - 1$ ,  $x^2$  is not present hence no coefficient. **Ans.**

**Q.3.** Give one example each of a binomial of degree 35, and of a monomial of degree 100.

**Sol.**  $x^{35} + 5$  is a binomial of degree 35.

$2y^{100}$  is a monomial of degree 100. **Ans.**

**Q.4.** Write the degree of each of the following polynomials :

(i)  $5x^3 + 4x^2 + 7x$  (ii)  $4 - y^2$  (iii)  $5t - \sqrt{7}$  (iv) 3

**Sol.** (i) Degree is 3 as  $x^3$  is the highest power. **Ans.**

(ii) Degree is 2 as  $y^2$  is the highest power. **Ans.**

(iii) Degree is 1 as  $t$  is the highest power. **Ans.**

(iv) Degree is 0 as  $x^0$  is the highest power. **Ans.**

**Q.5.** Classify the following as linear, quadratic and cubic polynomials :

(i)  $x^2 + x$  (ii)  $x - x^3$  (iii)  $y + y^2 + 4$   
(iv)  $1 + x$  (v)  $3t$  (vi)  $r^2$  (vii)  $7x^3$

**Sol.** (i)  $x^2 + x$  is quadratic. (ii)  $x - x^3$  is cubic.

(iii)  $y + y^2 + 4$  is quadratic. (iv)  $1 + x$  is linear.

(v)  $3t$  is linear. (vi)  $r^2$  is quadratic.

(vii)  $7x^3$  is cubic.

## EXERCISE 2.2

**Q.1.** Find the value of the polynomial  $5x - 4x^2 + 3$  at

- (i)  $x = 0$                       (ii)  $x = -1$                       (iii)  $x = 2$

**Sol.**  $p(x) = 5x - 4x^2 + 3$

(i) At  $x = 0$ ,  $p(0) = 5 \times 0 - 4 \times 0^2 + 3 = 3$  **Ans.**

(ii) At  $x = -1$ ,  $p(-1) = 5 \times (-1) - 4 \times (-1)^2 + 3 = -5 - 4 + 3 = -6$  **Ans.**

(iii) At  $x = 2$ ,  $p(2) = 5 \times 2 - 4 \times (2)^2 + 3 = 10 - 16 + 3 = -3$  **Ans.**

**Q.2.** Find  $p(0)$ ,  $p(1)$  and  $p(2)$  for each of the following polynomials :

(i)  $p(y) = y^2 - y + 1$                       (ii)  $p(t) = 2 + t + 2t^2 - t^3$

(iii)  $p(x) = x^3$                       (iv)  $p(x) = (x - 1)(x + 1)$

**Sol.** (i)  $p(y) = y^2 - y + 1$

$p(0) = 0^2 - 0 + 1 = 1$

$p(1) = 1^2 - 1 + 1 = 1$

$p(2) = 2^2 - 2 + 1 = 3$ . **Ans.**

(ii)  $p(t) = 2 + t + 2t^2 - t^3$

$p(0) = 2 + 0 + 2 \times 0^2 - 0^3 = 2$

$p(1) = 2 + 1 + 2 \times 1^2 - 1^3 = 4$

$p(2) = 2 + 2 + 2 \times 2^2 - 2^3 = 4 + 8 - 8 = 4$ .

(iii)  $p(x) = x^3$

$p(0) = 0$

$p(1) = 1$

$p(2) = 8$ . **Ans.**

(iv)  $p(x) = (x - 1)(x + 1)$

$p(0) = (-1)(1) = -1$

$p(1) = (1 - 1)(1 + 1) = 0$

$p(2) = (2 - 1)(2 + 1) = 3$  **Ans.**

**Q.3.** Verify whether the following are zeroes of the polynomial, indicated against them.

(i)  $p(x) = 3x + 1$ ,  $x = -\frac{1}{3}$                       (ii)  $p(x) = 5x - \pi$ ,  $x = \frac{4}{5}$

(iii)  $p(x) = x^2 - 1$ ,  $x = 1, -1$

(iv)  $p(x) = (x + 1)(x - 2)$ ,  $x = -1, 2$

(v)  $p(x) = x^2$ ,  $x = 0$

(vi)  $p(x) = lx + m$ ,  $x = -\frac{m}{l}$

(vii)  $p(x) = 3x^2 - 1$ ,  $x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii)  $p(x) = 2x + 1$ ,  $x = \frac{1}{2}$

**Sol.** (i) Yes.  $3x + 1 = 0$ , for  $x = -\frac{1}{3}$ . **Ans.**

(ii) No.  $5x - \pi = 5 \times \frac{4}{5} - \pi = 4 - \pi \neq 0$  **Ans.**

(iii) Yes.  $x^2 - 1 = 1^2 - 1 = 0$  for  $x = 1$

and  $x^2 - 1 = (-1)^2 - 1 = 0$  for  $x = -1$  **Ans.**

(iv) Yes.  $(x + 1)(x - 2) = 0$  for  $x = -1$ , or,  $x = 2$ .

(v) Yes.  $x^2 = 0$  for  $x = 0$

(vi) Yes.  $lx + m = 0$  for  $x = -\frac{m}{l}$

(vii)  $3x^2 - 1 = 3\frac{1}{3} - 1 = 0$  for  $x = \frac{-1}{\sqrt{3}}$

and  $3x^2 - 1 = 3 \cdot \frac{4}{3} - 1 = 3 \neq 0$

Thus, for  $\frac{-1}{\sqrt{3}}$  is a zero but  $-\frac{2}{\sqrt{3}}$  is not a zero of the polynomial **Ans.**

(viii) No.  $2x + 1 \neq 0$  for  $x = \frac{1}{2}$ .

**Q.4.** Find the zero of the polynomial in each of the following cases :

(i)  $p(x) = x + 5$

(ii)  $p(x) = x - 5$

(iii)  $p(x) = 2x + 5$

(iv)  $p(x) = 3x - 2$

(v)  $p(x) = 3x$

(vi)  $p(x) = ax, a \neq 0$

(vii)  $p(x) = cx + d, c \neq 0, c, d$  are real numbers.

**Sol.** (i)  $x + 5 = 0, x = -5$ , so,  $-5$  is the zero of  $x + 5$  **Ans.**

(ii)  $x - 5 = 0, x = 5$  so,  $5$  is the zero of  $x - 5$  **Ans.**

(iii)  $2x + 5 = 0, \Rightarrow 2x = -5,$

$\Rightarrow x = -\frac{5}{2}$ , so  $-\frac{5}{2}$  is the zero of  $2x + 5$  **Ans.**

(iv)  $3x - 2 = 0 \Rightarrow 3x = 2$

$\Rightarrow x = \frac{2}{3}$ , so  $\frac{2}{3}$  is the zero of  $3x - 2$  **Ans.**

(v)  $3x = 0, \Rightarrow x = 0$ , so  $0$  is the zero of  $3x$  **Ans.**

(vi)  $ax = 0 (a \neq 0) \Rightarrow x = \frac{0}{a} = 0$ , so,  $0$  is the zero of  $ax$  **Ans.**

(vii)  $cx + d = 0 (c \neq 0)$

$\Rightarrow cx = -d$

$\Rightarrow x = -\frac{d}{c}$ , so,  $-\frac{d}{c}$  is the 0 of  $cx + d$  **Ans.**

## EXERCISE 2.3

**Q.1.** Find the remainder when  $x^3 + 3x^2 + 3x + 1$  is divided by

(i)  $x + 1$                       (ii)  $x - \frac{1}{2}$                       (iii)  $x$

(iv)  $x + \pi$                       (v)  $5 + 2x$

**Sol.**  $p(x) = x^3 + 3x^2 + 3x + 1$

(i) When  $p(x)$  is divided by  $x + 1$ ,

i.e.,  $x + 1 = 0$ ,  $x = -1$  is to be substituted in  $p(x)$ .

$$p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$$

$$= -1 + 3 - 3 + 1 = 0$$

Remainder = 0. **Ans.**

(ii) When  $p(x)$  is divided by  $x - \frac{1}{2}$  remainder is  $p\left(\frac{1}{2}\right)$ .

$$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$$

$$= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1 = \frac{1+6+12+8}{8}$$

$$\therefore \text{Remainder} = \frac{27}{8} = 3\frac{3}{8} \text{ Ans.}$$

(iii) When  $p(x)$  is divided by  $x$ , then remainder is  $p(0)$ .

$x = 0$ , substitute in  $p(x)$

$$p(0) = 0^3 + 3 \times 0^2 + 3 \times 0 + 1 = 1.$$

$\therefore$  Remainder = 1 **Ans.**

(iv) When  $p(x)$  is divided by  $x + \pi$ , then, remainder is  $p(-\pi)$ .

$x = -\pi$  to be substituted in  $p(x)$

$$p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1.$$

$\therefore$  Remainder =  $-\pi^3 + 3\pi^2 - 3\pi + 1$  **Ans.**

(v) When  $p(x)$  is divided by  $(5 + 2x)$ , then remainder is  $p\left(\frac{-5}{2}\right)$ .

$$p\left(\frac{-5}{2}\right) = \left(\frac{-5}{2}\right)^3 + 3\left(\frac{-5}{2}\right)^2 + 3\left(\frac{-5}{2}\right) + 1$$

$$= \frac{-125}{8} + \frac{75}{4} - \frac{15}{2} + 1 = \frac{-125 + 150 - 60 + 8}{8}$$

$$\text{Remainder} = \frac{-35 + 8}{8} = \frac{-27}{8} \text{ Ans.}$$

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**Q.2.** Find the remainder when  $x^3 - ax^2 + 6x - a$  is divided by  $x - a$ .

**Sol.**  $p(x) = x^3 - ax^2 + 6x - a$

When  $p(x)$  is divided by  $x - a$ , the remainder is  $p(a)$ .

Substitute  $x = a$  in  $p(x)$

$$p(a) = a^3 - a^3 + 6a - a = 5a \quad \textbf{Ans.}$$

**Q.3.** Check whether  $7 + 3x$  is a factor of  $3x^3 + 7x$ .

**Sol.**  $7 + 3x = 0$

$$\Rightarrow 3x = -7$$

$$\Rightarrow x = \frac{-7}{3}$$

Substitute  $x = \frac{-7}{3}$  in  $p(x) = 3x^3 + 7x$

$$p\left(\frac{-7}{3}\right) = 3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right) = \frac{-343}{9} - \frac{49}{3} = \frac{-343 - 147}{9} = \frac{-490}{9}.$$

So, remainder =  $\frac{-490}{9}$  which is different from 0.

Therefore,  $(3x + 7)$  is not a factor of the polynomial  $3x^3 + 7x$ . **Ans.**

## EXERCISE 2.4

**Q.1.** Determine which of the following polynomials has  $(x + 1)$  a factor :

- (i)  $x^3 + x^2 + x + 1$  (ii)  $x^4 + x^3 + x^2 + x + 1$   
 (iii)  $x^4 + 3x^3 + 3x^2 + x + 1$  (iv)  $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

**Sol.** To have  $(x + 1)$  as a factor, substituting  $x = -1$  must give  $p(-1) = 0$ .

(i)  $x^3 + x^2 + x + 1$

$$= (-1)^3 + (-1)^2 + (-1) + 1 = -1 + 1 - 1 + 1 = 0$$

Therefore,  $x + 1$  is a factor of  $x^3 + x^2 + x + 1$  **Ans.**

(ii)  $x^4 + x^3 + x^2 + x + 1$

$$= (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1 = 1 - 1 + 1 - 1 + 1 = 1$$

Remainder is not 0. Therefore  $(x + 1)$  is not its factor. **Ans.**

(iii)  $x^4 + 3x^3 + 3x^2 + x + 1$

$$= (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$$

$$= 1 - 3 + 3 - 1 + 1 = 1. \text{ Remainder is not } 0$$

Therefore,  $(x + 1)$  is not its factor. **Ans.**

(iv)  $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

$$= (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$$

$$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

Remainder not 0, therefore  $(x + 1)$  is not a factor. **Ans.**

**Q.2.** Use the Factor Theorem to determine whether  $g(x)$  is a factor of  $p(x)$  in each of the following cases :

(i)  $p(x) = 2x^3 + x^2 - 2x - 1$ ,  $g(x) = x + 1$

(ii)  $p(x) = x^3 + 3x^2 + 3x + 1$ ,  $g(x) = x + 2$

(iii)  $p(x) = x^3 - 4x^2 + x + 6$ ,  $g(x) = x - 3$

**Sol.** (i)  $g(x) = x + 1$ .  $x = -1$  to be substituted in

$$p(x) = 2x^3 + x^2 - 2x - 1$$

$$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1 = -2 + 1 + 2 - 1 = 0.$$

So,  $g(x)$  is a factor of  $p(x)$ . **Ans.**

(ii)  $g(x) = x + 2$ , substitute  $x = -2$  in  $p(x)$

$$p(x) = x^3 + 3x^2 + 3x + 1$$

$$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1 = -8 + 12 - 6 + 1 = -1.$$

So,  $g(x)$  is not a factor of  $p(x)$  **Ans.**

(iii)  $g(x) = x - 3$  substitute  $x = 3$  in  $p(x)$ .

$$p(x) = x^3 - 4x^2 + x + 6$$

$$p(3) = (3)^3 - 4(3)^2 + 3 + 6 = 27 - 36 + 3 + 6 = 0.$$

Therefore,  $g(x)$  is a factor of  $x^3 - 4x^2 + x + 6$ . **Ans.**

**Q.3.** Find the value of  $k$ , if  $x - 1$  is a factor of  $p(x)$  in each of the following cases :

(i)  $p(x) = x^2 + x + k$                       (ii)  $p(x) = 2x^2 + kx + \sqrt{2}$

(iii)  $p(x) = kx^2 - \sqrt{2}x + 1$                       (iv)  $p(x) = kx^2 - 3x + k$

**Sol.**  $(x - 1)$  is a factor, so we substitute  $x = 1$  in each case and solve for  $k$  by making  $p(1)$  equal to 0.

(i)  $p(x) = x^2 + x + k$   
 $p(1) = 1 + 1 + k = 0 \Rightarrow k = -2$  **Ans.**

(ii)  $p(x) = 2x^2 + kx + \sqrt{2}$   
 $p(1) = 2 \times 1^2 + k \times 1 + \sqrt{2} = 0$   
 $\Rightarrow 2 + k + \sqrt{2} = 0$   
 $\Rightarrow k = -2 - \sqrt{2} = -(2 + \sqrt{2})$  **Ans.**

(iii)  $p(x) = kx^2 - \sqrt{2}x + 1$   
 $p(1) = k - \sqrt{2} + 1 = 0$   
 $\Rightarrow k = \sqrt{2} - 1$  **Ans.**

(iv)  $p(x) = kx^2 - 3x + k$   
 $p(1) = k - 3 + k = 0 \Rightarrow 2k - 3 = 0$   
 $\Rightarrow k = \frac{3}{2}$  **Ans.**

**Q.4.** Factorise :

(i)  $12x^2 - 7x + 1$     (ii)  $2x^2 + 7x + 3$     (iii)  $6x^2 + 5x - 6$     (iv)  $3x^2 - x - 4$

**Sol.** (i)  $12x^2 - 7x + 1$   
 $= 12x^2 - 4x - 3x + 1$   
 $= 4x(3x - 1) - 1(3x - 1) = (4x - 1)(3x - 1)$  **Ans.**

(ii)  $2x^2 + 7x + 3$   
 $= 2x^2 + 6x + x + 3$   
 $= 2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$  **Ans.**

(iii)  $6x^2 + 5x - 6$   
 $= 6x^2 + 9x - 4x - 6$   
 $= 3x(2x + 3) - 2(2x + 3) = (3x - 2)(2x + 3)$  **Ans.**

(iv)  $3x^2 - x - 4$   
 $= 3x^2 - 4x + 3x - 4 = x(3x - 4) + 1(3x - 4) = (x + 1)(3x - 4)$  **Ans.**

**Q.5.** Factorise :

(i)  $x^3 - 2x^2 - x + 2$                       (ii)  $x^3 - 3x^2 - 9x - 5$

(iii)  $x^3 + 13x^2 + 32x + 20$                       (iv)  $2y^3 + y^2 - 2y - 1$

**Sol.** (i)  $p(x) = x^3 - 2x^2 - x + 2$

Let us guess a factor  $(x - a)$  and choose value of  $a$  arbitrarily as 1.  
Now, putting this value in  $p(x)$ .

$$1 - 2 - 1 + 2 = 0$$

So  $(x - 1)$  is a factor of  $p(x)$

$$\begin{aligned} \text{Now, } x^3 - 2x^2 - x + 2 &= x^3 - x^2 - x^2 + x - 2x + 2 \\ &= x^2(x - 1) - x(x - 1) - 2(x - 1) \\ &= (x - 1)(x^2 - x - 2) \\ &= (x - 1)(x^2 - 2x + x - 2) \\ &= (x - 1)\{x(x - 2) + 1(x - 2)\} \\ &= (x - 1)(x + 1)(x - 2) \text{ **Ans.**} \end{aligned}$$

To factorise it

$$x^2 - 2x + x - 2 = x(x - 2) + 1(x - 2) = (x + 1)(x - 2).$$

After factorisation  $(x - 1)(x + 1)(x - 2)$ .

(ii)  $p(x) = x^3 - 3x^2 - 9x - 5$

Take a factor  $(x - a)$ .  $a$  should be a factor of 5, i.e.,  $\pm 1$  or  $\pm 5$ .

For  $(x - 1)$ ,  $a = 1$

$$\begin{aligned} p(1) &= (1)^3 - (-3) 1^2 - 9 \times 1 - 5 \\ &= 1 - 3 - 9 - 5 = -16. \end{aligned}$$

So,  $(x - 1)$  is not a factor of  $p(x)$ .

For  $a = 5$

$$\begin{aligned} p(5) &= (5)^3 - 3(5)^2 - 9(5) - 5 \\ &= 125 - 75 - 45 - 5 = 0. \end{aligned}$$

Therefore,  $(x - 5)$  is a factor of  $x^3 - 3x^2 - 9x - 5$ .

$$\begin{aligned} \text{Now, } x^3 - 3x^2 - 9x - 5 &= x^3 - 5x^2 + 2x^2 - 10x + x - 5 \\ &= x^2(x - 5) + 2x(x - 5) + 1(x - 5) \\ &= (x - 5)(x^2 + 2x + 1) \\ &= (x - 5)(x + 1)^2 \\ &= (x - 5)(x + 1)(x + 1) \end{aligned}$$

So,  $x^3 - 3x^2 - 9x - 5 = (x - 5)(x + 1)(x + 1)$ . **Ans.**

(iii)  $p(x) = x^3 + 13x^2 + 32x + 20$

Let a factor be  $(x - a)$ .  $a$  should be a factor of 20 which are  $\pm 1$ ,  $\pm 2$ ,  $\pm 4$ ,  $\pm 5$ ,  $\pm 10$ .

For  $x - 1 = 0 \Rightarrow x = 1$

$$\begin{aligned} \text{Now, } p(1) &= 1 + 13 + 32 + 20 \\ &= 66 \neq 0 \end{aligned}$$

Hence,  $(x - 1)$  is not a factor of  $p(x)$ .

Again, for  $x + 1 = 0 \Rightarrow x = -1$

$$\begin{aligned} \text{Now, } p(-1) &= -1 + 13 - 32 + 20 \\ &= -33 + 33 = 0 \end{aligned}$$

Hence,  $(x + 1)$  is a factor of  $p(x)$ .

$$\begin{aligned} \text{Now, } x^3 + 13x^2 + 32x + 20 &= x^3 + x^2 + 12x^2 + 20x + 20 \\ &= x^2(x + 1) + 12x(x + 1) + 20(x + 1) \\ &= (x + 1)(x^2 + 12x + 20) \\ &= (x + 1)(x^2 + 10x + 2x + 20) \\ &= (x + 1)\{x(x + 10) + 2(x + 10)\} \\ &= (x + 1)(x + 1)(x + 10) \quad \text{Ans.} \end{aligned}$$

(iv)  $p(y) = 2y^3 + y^2 - 2y - 1$

factors of  $-1$  are  $\pm 1$ ,  $\pm 1$ .

$$\begin{aligned} p(1) &= 2 \times 1^3 + 1^2 - 2 \times 1 - 1 \\ &= 2 + 1 - 2 - 1 = 0. \end{aligned}$$

Therefore,  $(y - 1)$  is a factor of  $p(y)$ .

$$\begin{aligned} \text{Now, } 2y^3 + y^2 - 2y - 1 &= 2y^3 - 2y^2 + 3y^2 - 3y + y - 1 \\ &= 2y^2(y - 1) + 3y(y - 1) + 1(y - 1) \\ &= (y - 1)(2y^2 + 3y + 1) \\ &= (y - 1)(2y^2 + 2y + y + 1) \\ &= (y - 1)\{2y(y + 1) + 1(y + 1)\} \\ &= (y - 1)(y + 1)(2y + 1) \end{aligned}$$

Therefore,  $2y^3 + y^2 - 2y - 1 = (y - 1)(y + 1)(2y + 1)$ . **Ans.**



## EXERCISE 2.5

**Q.1.** Use suitable identities to find the following products :

- (i)  $(x + 4)(x + 10)$       (ii)  $(x + 8)(x - 10)$   
 (iii)  $(3x + 4)(3x - 5)$       (iv)  $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$       (v)  $(3 - 2x)(3 + 2x)$

- Sol.** (i) Using identity  $(x + a)(x + b) = x^2 + (a + b)x + ab$   
 $(x + 4)(x + 10) = x^2 + (4 + 10)x + 4 \times 10 = x^2 + 14x + 40$  **Ans.**  
 (ii) Using the same identity as in (i) above  
 $(x + 8)(x - 10) = x^2 + (8 - 10)x + 8 \times (-10) = x^2 - 2x - 80$  **Ans.**  
 (iii) Using the same identity  
 $(3x + 4)(3x - 5) = 3x \times 3x + (-1)(3x) - 20 = 9x^2 - 3x - 20$  **Ans.**  
 (iv) Using  $(x + y)(x - y) = x^2 - y^2$   
 $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) = (y^2)^2 - \left(\frac{3}{2}\right)^2 = y^4 - \frac{9}{4}$  **Ans.**  
 (v) Using the same identity as in (iv)  
 $(3 - 2x)(3 + 2x) = 3^2 - (2x)^2$   
 $= 9 - 4x^2$  **Ans.**

**Q.2.** Evaluate the following products without multiplying directly :

- (i)  $103 \times 107$       (ii)  $95 \times 96$       (iii)  $104 \times 96$

- Sol.** (i)  $103 \times 107 = (100 + 3)(100 + 7)$   
 $= (100)^2 + (3 + 7) \times 100 + 3 \times 7$   
 $= 10000 + 1000 + 21 = 11021$  **Ans.**  
 (ii)  $95 \times 96 = (100 - 5)(100 - 4)$   
 $= (100)^2 - (5 + 4) \times 100 + 5 \times 4$   
 $= 10000 - 900 + 20 = 9120$  **Ans.**  
 (iii)  $104 \times 96 = (100 + 4)(100 - 4) = 100^2 - 4^2$   
 $= 10000 - 16 = 9984$  **Ans.**

**Q.3.** Factorise the following using appropriate identities :

(i)  $9x^2 + 6xy + y^2$     (ii)  $4y^2 - 4y + 1$     (iii)  $x^2 - \frac{y^2}{100}$

**Sol.** (i)  $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)y + (y)^2$   
 $= (3x + y)^2$  [Using  $a^2 + 2ab + b^2 = (a + b)^2$ ]

$= (3x + y)(3x + y)$  **Ans.**

(ii)  $4y^2 - 4y + 1$   
 $= (2y)^2 - 2(2y)(1) + (1)^2$   
 $= (2y - 1)^2 = (2y - 1)(2y - 1)$  [Using  $a^2 - 2ab + b^2 = (a - b)^2$ ] **Ans.**

(iii)  $x^2 - \frac{y^2}{100} = x^2 - \left(\frac{y}{10}\right)^2$   
 $= \left(\frac{x + y}{10}\right)\left(\frac{x - y}{10}\right)$  [Using  $a^2 - b^2 = (a + b)(a - b)$ ] **Ans.**

**Q.4.** Expand each of the following, using suitable identities :

(i)  $(x + 2y + 4z)^2$     (ii)  $(2x - y + z)^2$     (iii)  $(-2x + 3y + 2z)^2$

(iv)  $(3a - 7b - c)^2$     (v)  $(-2x + 5y - 3z)^2$     (vi)  $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

**Sol.** (i)  $(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2x \times 2y + 2 \times 2y \times 4z + 2 \times 4z \times x$   
 $= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$  **Ans.**

(ii)  $(2x - y + z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2 \times (2x)(-y)$   
 $+ 2(-y)(z) + 2(z) \times 2x$   
 $= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$  **Ans.**

(iii)  $(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y)$   
 $+ 2(3y)(2z) + 2(2z)(-2x)$   
 $= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$  **Ans.**

(iv)  $(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b)$   
 $+ 2(-7b)(-c) + 2(-c)(3a)$   
 $= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$  **Ans.**

(v)  $(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y)$   
 $+ 2(5y)(-3z) + 2(-3z)(-2x)$   
 $= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$  **Ans.**

$$\begin{aligned}
 \text{(vi)} \quad \left[ \frac{1}{4}a - \frac{1}{2}b + 1 \right]^2 &= \left( \frac{1}{4}a \right)^2 + \left( \frac{-1}{2}b \right)^2 + (1)^2 + 2 \left( \frac{1}{4}a \right) \left( \frac{-1}{2}b \right) \\
 &\quad + 2 \left( \frac{-1}{2}b \right) \times 1 + 2(1) \times \frac{1}{4}a \\
 &= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a \\
 &= \frac{a^2}{16} + \frac{b^2}{4} + 1 - \frac{ab}{4} - b + \frac{a}{2} \quad \text{Ans.}
 \end{aligned}$$

**Q.5.** Factorise :

(i)  $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii)  $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

**Sol.** (i)  $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$   
 $= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$   
 $= (2x + 3y - 4z)^2 = (2x + 3y - 4z)(2x + 3y - 4z) \quad \text{Ans.}$

(ii)  $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$   
 $= (\sqrt{2}x)^2 + (-y)^2 + (-2\sqrt{2}z)^2 + 2(\sqrt{2}x)(-y) + 2(-y)(-2\sqrt{2}z)$   
 $\quad + 2(\sqrt{2}x)(-2\sqrt{2}z)$   
 $= (\sqrt{2}x - y - 2\sqrt{2}z)^2$   
 $= (\sqrt{2}x - y - 2\sqrt{2}z)(\sqrt{2}x - y - 2\sqrt{2}z) \quad \text{Ans.}$

**Q.6.** Write the following cubes in expanded form :

(i)  $(2x + 1)^3$     (ii)  $(2a - 3b)^3$     (iii)  $\left[ \frac{3}{2}x + 1 \right]^3$     (iv)  $\left[ x - \frac{2}{3}y \right]^3$

**Sol.** (i)  $(2x + 1)^3 = (2x)^3 + 1^3 + 3(2x)(1)(2x + 1)$   
 $= 8x^3 + 1 + 6x(2x + 1) = 8x^3 + 12x^2 + 6x + 1 \quad \text{Ans.}$

(ii)  $(2a - 3b)^3 = (2a)^3 - (3b)^3 - 3 \times 2a \times 3b(2a - 3b)$   
 $= 8a^3 - 27b^3 - 18ab(2a - 3b)$   
 $= 8a^3 - 27b^3 - 36a^2b + 54ab^2 \quad \text{Ans.}$

(iii)  $\left[ \frac{3}{2}x + 1 \right]^3 = \left( \frac{3}{2}x \right)^3 + 1^3 + 3 \left( \frac{3}{2}x \right)(1) \left( \frac{3}{2}x + 1 \right)$   
 $= \frac{27}{8}x^3 + 1 + \frac{9}{2}x \left( \frac{3}{2}x + 1 \right)$   
 $= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1 \quad \text{Ans.}$

$$\begin{aligned}
 \text{(iv)} \quad \left[ x - \frac{2}{3}y \right]^3 &= x^3 - \left( \frac{2}{3}y \right)^3 - 3(x) \left( \frac{2}{3}y \right) \left( x - \frac{2}{3}y \right) \\
 &= x^3 - \frac{8}{27}y^3 - 2xy \left( x - \frac{2}{3}y \right) \\
 &= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2 \quad \text{Ans.}
 \end{aligned}$$

**Q.7.** Evaluate the following using suitable identities :

$$(i) (99)^3 \qquad (ii) (102)^3 \qquad (iii) (998)^3$$

**Sol.**

$$\begin{aligned}
 (i) \quad (99)^3 &= (100 - 1)^3 = (100)^3 + (-1)^3 + 3(100)(-1)(100 - 1) \\
 &= 1000000 - 1 - 300(100 - 1) \\
 &= 1000000 - 1 - 30000 + 300 = 970299 \\
 (ii) \quad (102)^3 &= (100 + 2)^3 = 100^3 + 2^3 + 3(100)(2)(100 + 2) \\
 &= 1000000 + 8 + 600(100 + 2) \\
 &= 1000000 + 8 + 60000 + 1200 = 1061208 \quad \text{Ans.} \\
 (iii) \quad (998)^3 &= (1000 - 2)^3 = (1000)^3 + (-2)^3 + 3(1000)(-2)(998) \\
 &= (1000)^3 - 8 - 6000(998) \\
 &= 1000000000 - 8 - 5988000 = 994011992 \quad \text{Ans.}
 \end{aligned}$$

**Q.8.** Factorise each of the following :

$$\begin{aligned}
 (i) & 8a^3 + b^3 + 12a^2b + 6ab^2 & (ii) & 8a^3 - b^3 - 12a^2b + 6ab^2 \\
 (iii) & 27 - 125a^3 - 135a + 225a^2 \\
 (iv) & 64a^3 - 27b^3 - 144a^2b + 108ab^2 \\
 (v) & 27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p
 \end{aligned}$$

**Sol.**

$$\begin{aligned}
 (i) \quad 8a^3 + b^3 + 12a^2b + 6ab^2 &= (2a)^3 + b^3 + 3(2a)(b)(2a + b) \\
 &= (2a + b)^3 = (2a + b)(2a + b)(2a + b) \quad \text{Ans.} \\
 (ii) \quad 8a^3 - b^3 - 12a^2b + 6ab^2 &= (2a)^3 + (-b)^3 + 3(2a)(-b)(2a - b) \\
 &= (2a - b)^3 = (2a - b)(2a - b)(2a - b) \quad \text{Ans.} \\
 (iii) \quad 27 - 125a^3 - 135a + 225a^2 &= 3^3 + (-5a)^3 + 3 \times (3)(-5a)(3 - 5a) \\
 &= (3 - 5a)^3 = (3 - 5a)(3 - 5a)(3 - 5a) \quad \text{Ans.} \\
 (iv) \quad 64a^3 - 27b^3 - 144a^2b + 108ab^2 &= (4a)^3 + (-3b)^3 + 3(4a) \times (-3b)(4a - 3b) \\
 &= (4a - 3b)^3 = (4a - 3b)(4a - 3b)(4a - 3b) \quad \text{Ans.} \\
 (v) \quad 27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p &= (3p)^3 + \left( -\frac{1}{6} \right)^3 + 3(3p) \left( -\frac{1}{6} \right) \left( 3p - \frac{1}{6} \right) \\
 &= \left( 3p - \frac{1}{6} \right)^3 = \left( 3p - \frac{1}{6} \right) \left( 3p - \frac{1}{6} \right) \left( 3p - \frac{1}{6} \right) \quad \text{Ans.}
 \end{aligned}$$

**Q.9.** Verify : (i)  $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii)  $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

**Sol.** (i)  $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$$\text{R.H.S.} = x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$$

$$= x^3 - x^2y + xy^2 + yx^2 - xy^2 + y^3 = x^3 + y^3 = \text{L.H.S.}$$

(ii)  $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$$\text{R.H.S.} = x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$$

$$= x^3 + x^2y + xy^2 - yx^2 - xy^2 - y^3 = x^3 - y^3 = \text{L.H.S.}$$

**Q.10.** Factorise each of the following :

(i)  $27y^3 + 125z^3$

(ii)  $64m^3 - 343n^3$

**Sol.** (i)  $27y^3 + 125z^3 = (3y)^3 + (5z)^3 = (3y + 5z)[(3y)^2 - (3y)(5z) + (5z)^2]$   
 $= (3y + 5z)(9y^2 - 15yz + 25z^2)$  **Ans.**

(ii)  $64m^3 - 343n^3 = (4m)^3 - (7n)^3$

$$= (4m - 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$$

$$= (4m - 7n)(16m^2 + 28mn + 49n^2)$$

**Q.11.** Factorise :  $27x^3 + y^3 + z^3 - 9xyz$

**Sol.**  $27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3(3x)yz$

$$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx) \quad \text{Ans.}$$

**Q.12.** Verify that :

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

**Sol.** To verify :

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

$$\text{R.H.S.} = \frac{1}{2}(x + y + z)[x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + z^2 + x^2 - 2zx]$$

$$= \frac{1}{2}(x + y + z)[2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx]$$

$$= (x + y + z)[x^2 + y^2 + z^2 - xy - yz - zx]$$

$$= x[x^2 + y^2 + z^2 - xy - yz - zx]$$

$$+ y(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$+ z(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= x^3 + xy^2 + xz^2 - x^2y - xyz - zx^2 + yx^2 + y^3 + yz^2 - xy^2$$

$$- y^2z - zxy + zx^2 + zy^2 + z^3 - zxy - yz^2 - z^2x$$

$$= x^3 + y^3 + z^3 - 3xyz = \text{L.H.S.} \quad \text{Hence verified.}$$

**Q.13.** If  $x + y + z = 0$ , show that  $x^3 + y^3 + z^3 = 3xyz$ .

**Sol.**  $x + y + z = 0$

$$(x + y + z)^3 = x^3 + y^3 + z^3 - 3xyz = 0$$

$$\Rightarrow x^3 + y^3 + z^3 = 3xyz. \quad \text{Proved.}$$

**Q.14.** Without actually calculating the cubes, find the value of each of the following :

(i)  $(-12)^3 + (7)^3 + (5)^3$

(ii)  $(28)^3 + (-15)^3 + (-13)^3$

**Sol.** From the above question, we have  $x^3 + y^3 + z^3 = 3xyz$ , if  $x + y + z = 0$

(i) Here  $-12 + 7 + 5 = 0$

$$(-12)^3 + (7)^3 + (5)^3$$

$$= 3(-12)(7)(5) = -1260 \quad \text{Ans.}$$

(ii) Here  $28 + (-15) + (-13) = 0$   
 So,  $(28)^3 + (-15)^3 + (-13)^3$   
 $= 3 \times 28 (-15) (-13) = 16380$  **Ans.**

**Q.15.** Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given :

$$\boxed{\text{Area : } 25a^2 - 35a + 12}$$

(i)

$$\boxed{\text{Area : } 35y^2 + 13y - 12}$$

(ii)

**Sol.** (i)  $\text{Area} = 25a^2 - 35a + 12$   
 $= 25a^2 - 20a - 15a + 12$   
 $= 5a (5a - 4) - 3 (5a - 4)$   
 $= (5a - 4) (5a - 3)$

So, one possible answer is length =  $(5a - 4)$ , breadth =  $(5a - 3)$

Therefore  $p\left(\frac{3}{5}\right)$  gives zero value and  $(5a - 3)$  is a factor.

Second factor  $(5a - 4)$ , length =  $(5a - 3)$ ; breadth =  $(5a - 4)$ .

(ii)  $\text{Area} = 35y^2 + 13y - 12$   
 $= 35y^2 + 28y - 15y - 12$   
 $= 7y (5y + 4) - 3 (5y + 4)$   
 $= (5y + 4) (7y - 3)$

So,  $(5y + 4)$  may be taken as breadth and  $(7y - 3)$  as length. **Ans.**

**Q.16.** What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

$$\boxed{\text{Volume : } 3x^2 - 12x}$$

(i)

$$\boxed{\text{Volume : } 12ky^2 + 8ky - 20k}$$

(ii)

**Sol.** (i)  $abc = 3x^2 - 12x = 3x (x - 4)$

$3, x (x - 4)$  are the three factors so they can be three dimensions.

(ii)  $abc = 12ky^2 + 8ky - 20k$   
 $= 4k (3y^2 + 2y - 5)$   
 $= 4k \{3y (y - 1) + 5 (y - 1)\}$   
 $= 4k (y - 1) (3y + 5)$

$4k, (y - 1)$  and  $(3y + 5)$  are the three factors, so they can be three dimensions **Ans.**