

# CHAPTER - 8

## QUADRILATERALS

### EXERCISE 8.1

**Q.1.** The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

**Sol.** Suppose the measures of four angles are  $3x$ ,  $5x$ ,  $9x$  and  $13x$ .

$$\therefore 3x + 5x + 9x + 13x = 360^\circ \quad [\text{Angle sum property of a quadrilateral}]$$

$$\Rightarrow 30x = 360^\circ$$

$$\Rightarrow x = \frac{360^\circ}{30} = 12^\circ$$

$$\Rightarrow 3x = 3 \times 12^\circ = 36^\circ$$

$$5x = 5 \times 12^\circ = 60^\circ$$

$$9x = 9 \times 12^\circ = 108^\circ$$

$$13x = 13 \times 12^\circ = 156^\circ$$

$\therefore$  the angles of the quadrilateral are  **$36^\circ$ ,  $60^\circ$ ,  $108^\circ$  and  $156^\circ$**  Ans.

**Q.2.** If the diagonals of a parallelogram are equal, then show that it is a rectangle.

**Sol. Given :** ABCD is a parallelogram in which  $AC = BD$ .

**To Prove :** ABCD is a rectangle.

**Proof :** In  $\triangle ABC$  and  $\triangle BAD$

$$AB = AB \quad [\text{Common}]$$

$$BC = AD \quad [\text{Opposite sides of a parallelogram}]$$

$$AC = BD \quad [\text{Given}]$$

$$\therefore \triangle ABC \cong \triangle BAD \quad [\text{SSS congruence}]$$

$$\angle ABC = \angle BAD \quad \dots(i) \quad [\text{CPCT}]$$

Since, ABCD is a parallelogram, thus,

$$\angle ABC + \angle BAD = 180^\circ \quad \dots(ii)$$

[Consecutive interior angles]

$$\angle ABC + \angle ABC = 180^\circ$$

$$\therefore 2\angle ABC = 180^\circ \quad [\text{From (i) and (ii)}]$$

$$\Rightarrow \angle ABC = \angle BAD = 90^\circ$$

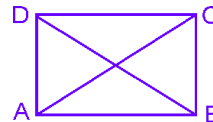
This shows that ABCD is a parallelogram one of whose angle is  $90^\circ$ .

Hence, ABCD is a rectangle. **Proved.**

**Q.3.** Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

**Sol. Given :** A quadrilateral ABCD, in which diagonals AC and BD bisect each other at right angles.

**To Prove :** ABCD is a rhombus.



**Proof :** In  $\triangle AOB$  and  $\triangle BOC$

$$AO = OC$$

[Diagonals AC and BD bisect each other]

$$\angle AOB = \angle COB \quad [\text{Each} = 90^\circ]$$

$$BO = BO \quad [\text{Common}]$$

$$\therefore \triangle AOB \cong \triangle BOC \quad [\text{SAS congruence}]$$

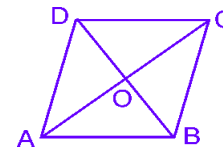
$$AB = BC \quad \dots(i) \quad [\text{CPCT}]$$

Since, ABCD is a quadrilateral in which

$$AB = BC \quad [\text{From (i)}]$$

Hence, ABCD is a rhombus.

[ $\therefore$  if the diagonals of a quadrilateral bisect each other, then it is a parallelogram and opposite sides of a parallelogram are equal] **Proved.**



**Q.4.** Show that the diagonals of a square are equal and bisect each other at right angles.

**Sol. Given :** ABCD is a square in which AC and BD are diagonals.

**To Prove :** AC = BD and AC bisects BD at right angles, i.e.  $AC \perp BD$ .

$$AO = OC, OB = OD$$

**Proof :** In  $\triangle ABC$  and  $\triangle BAD$ ,

$$AB = AB \quad [\text{Common}]$$

$$BC = AD \quad [\text{Sides of a square}]$$

$$\angle ABC = \angle BAD = 90^\circ \quad [\text{Angles of a square}]$$

$$\therefore \triangle ABC \cong \triangle BAD \quad [\text{SAS congruence}]$$

$$\Rightarrow AC = BD \quad [\text{CPCT}]$$

Now in  $\triangle AOB$  and  $\triangle COD$ ,

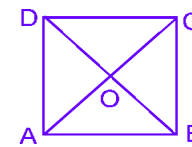
$$AB = DC \quad [\text{Sides of a square}]$$

$$\angle AOB = \angle COD \quad [\text{Vertically opposite angles}]$$

$$\angle OAB = \angle OCD \quad [\text{Alternate angles}]$$

$$\therefore \triangle AOB \cong \triangle COD \quad [\text{AAS congruence}]$$

$$\angle AO = \angle OC \quad [\text{CPCT}]$$



Similarly by taking  $\triangle AOD$  and  $\triangle BOC$ , we can show that  $OB = OD$ .

$$\text{In } \triangle ABC, \angle BAC + \angle BCA = 90^\circ \quad [ \because \angle B = 90^\circ ]$$

$$\Rightarrow 2\angle BAC = 90^\circ \quad [\angle BAC = \angle BCA, \text{ as } BC = AD]$$

$$\Rightarrow \angle BCA = 45^\circ \quad \text{or } \angle BCO = 45^\circ$$

$$\text{Similarly } \angle CBO = 45^\circ$$

In  $\triangle BCO$ .

$$\angle BCO + \angle CBO + \angle BOC = 180^\circ$$

$$\Rightarrow 90^\circ + \angle BOC = 180^\circ$$

$$\Rightarrow \angle BOC = 90^\circ$$

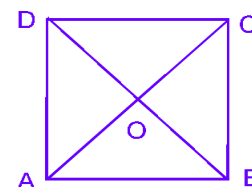
$$\Rightarrow BO \perp OC \Rightarrow BO \perp AC$$

Hence,  $AC = BD$ ,  $AC \perp BD$ ,  $AO = OC$  and  $OB = OD$ . **Proved.**

**Q.5.** Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

**Sol. Given :** A quadrilateral ABCD, in which diagonals AC and BD are equal and bisect each other at right angles,

**To Prove :** ABCD is a square.



**Proof :** Since ABCD is a quadrilateral whose diagonals bisect each other, so it is a parallelogram. Also, its diagonals bisect each other at right angles, therefore, ABCD is a rhombus.

$$\Rightarrow AB = BC = CD = DA \quad \text{[Sides of a rhombus]}$$

In  $\triangle ABC$  and  $\triangle BAD$ , we have

$$AB = AB \quad \text{[Common]}$$

$$BC = AD \quad \text{[Sides of a rhombus]}$$

$$AC = BD \quad \text{[Given]}$$

$$\therefore \triangle ABC \cong \triangle BAD \quad \text{[SSS congruence]}$$

$$\therefore \angle ABC = \angle BAD \quad \text{[CPCT]}$$

$$\text{But, } \angle ABC + \angle BAD = 180^\circ \quad \text{[Consecutive interior angles]}$$

$$\angle ABC = \angle BAD = 90^\circ$$

$$\angle A = \angle B = \angle C = \angle D = 90^\circ \quad \text{[Opposite angles of a } \parallel^{\text{gm}} \text{]}$$

$\Rightarrow$  ABCD is a rhombus whose angles are of  $90^\circ$  each.

Hence, ABCD is a square. **Proved.**

**Q.6.** Diagonal AC of a parallelogram ABCD bisects  $\angle A$  (see Fig.). Show that

(i) it bisects  $\angle C$  also,

(ii) ABCD is a rhombus.

**Given :** A parallelogram ABCD, in which diagonal AC bisects  $\angle A$ , i.e.,  $\angle DAC = \angle BAC$ .

**To Prove :** (i) Diagonal AC bisects  $\angle C$  i.e.,  $\angle DCA = \angle BCA$

(ii) ABCD is a rhombus.

**Proof :** (i)  $\angle DAC = \angle BCA$

$$\angle BAC = \angle DCA$$

$$\text{But, } \angle DAC = \angle BAC$$

$$\therefore \angle BCA = \angle DCA$$

Hence, AC bisects  $\angle DCB$

Or, AC bisects  $\angle C$  **Proved.**

(ii) In  $\triangle ABC$  and  $\triangle CDA$

$$AC = AC \quad \text{[Common]}$$

$$\angle BAC = \angle DAC \quad \text{[Given]}$$

$$\text{and } \angle BCA = \angle DCA \quad \text{[Proved above]}$$

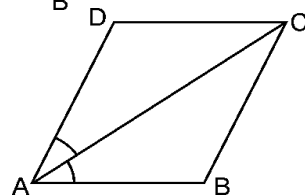
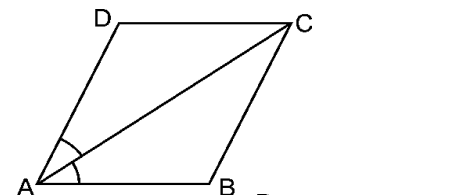
$$\therefore \triangle ABC \cong \triangle ADC \quad \text{[ASA congruence]}$$

$$\therefore BC = DC \quad \text{[CPCT]}$$

$$\text{But } AB = DC \quad \text{[Given]}$$

$$\therefore AB = BC = DC = AD$$

Hence, ABCD is a rhombus **Proved.**



[Alternate angles]

[Alternate angles]

[Given]

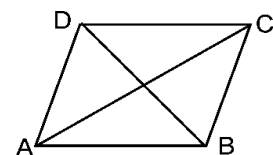
[ $\therefore$  opposite angles are equal]

**Q.7.** ABCD is a rhombus. Show that diagonal AC bisects  $\angle A$  as well as  $\angle C$  and diagonal BD bisects  $\angle B$  as well as  $\angle D$ .

**Sol. Given :** ABCD is a rhombus, i.e.,

$$AB = BC = CD = DA.$$

**To Prove :**  $\angle DAC = \angle BAC$ ,



$$\angle BCA = \angle DCA$$

$$\angle ADB = \angle CDB, \angle ABD = \angle CBD$$

**Proof :** In  $\triangle ABC$  and  $\triangle CDA$ , we have

$$AB = AD \quad \text{[Sides of a rhombus]}$$

$$AC = AC \quad \text{[Common]}$$

$$BC = CD \quad \text{[Sides of a rhombus]}$$

$$\triangle ABC \cong \triangle ADC \quad \text{[SSS congruence]}$$

$$\text{So, } \left. \begin{array}{l} \angle DAC = \angle BAC \\ \angle BCA = \angle DCA \end{array} \right\} \text{ [CPCT]}$$

Similarly,  $\angle ADB = \angle CDB$  and  $\angle ABD = \angle CBD$ .

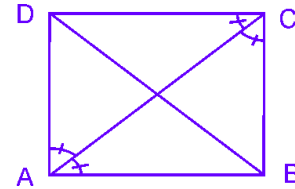
Hence, diagonal AC bisects  $\angle A$  as well as  $\angle C$  and diagonal BD bisects  $\angle B$  as well as  $\angle D$ . **Proved.**

**Q.8.** ABCD is a rectangle in which diagonal AC bisects  $\angle A$  as well as  $\angle C$ . Show that :

(i) ABCD is a square (ii) diagonal BD bisects  $\angle B$  as well as  $\angle D$ .

**Sol. Given :** ABCD is a rectangle in which diagonal AC bisects  $\angle A$  as well as  $\angle C$ .

**To Prove :** (i) ABCD is a square.  
(ii) Diagonal BD bisects  $\angle B$  as well as  $\angle D$ .



**Proof :** (i) In  $\triangle ABC$  and  $\triangle ADC$ , we have  
 $\angle BAC = \angle DAC$  [Given]  
 $\angle BCA = \angle DCA$  [Given]  
 $AC = AC$

$$\therefore \triangle ABC \cong \triangle ADC \quad \text{[ASA congruence]}$$

$$\therefore AB = AD \text{ and } CB = CD \quad \text{[CPCT]}$$

But, in a rectangle opposite sides are equal,

$$\text{i.e., } AB = DC \text{ and } BC = AD$$

$$\therefore AB = BC = CD = DA$$

Hence, ABCD is a square **Proved.**

(ii) In  $\triangle ABD$  and  $\triangle CDB$ , we have

$$\left. \begin{array}{l} AD = CD \\ AB = CB \end{array} \right\} \text{ [Sides of a square]}$$

$$BD = BD \quad \text{[Common]}$$

$$\therefore \triangle ABD \cong \triangle CDB \quad \text{[SSS congruence]}$$

$$\text{So, } \left. \begin{array}{l} \angle ABD = \angle CBD \\ \angle ADB = \angle CDB \end{array} \right\} \text{ [CPCT]}$$

Hence, diagonal BD bisects  $\angle B$  as well as  $\angle D$  **Proved.**

**Q.9.** In parallelogram ABCD, two points P and Q are taken on diagonal BD such that  $DP = BQ$  (see Fig.). Show that :

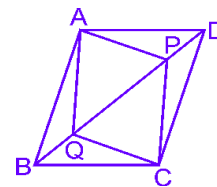
(i)  $\triangle APD \cong \triangle CQB$

(ii)  $AP = CQ$

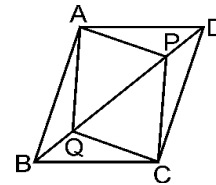
(iii)  $\triangle AQB \cong \triangle CPD$

(iv)  $AQ = CP$

(v) APCQ is a parallelogram



**Sol. Given :** ABCD is a parallelogram and P and Q are points on diagonal BD such that DP = BQ.

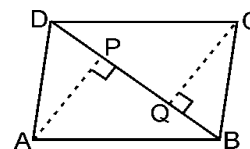


- To Prove :**
- (i)  $\triangle APD \cong \triangle CQB$
  - (ii)  $AP = CQ$
  - (iii)  $\triangle AQB \cong \triangle CPD$
  - (iv)  $AQ = CP$
  - (v) APCQ is a parallelogram

**Proof :**

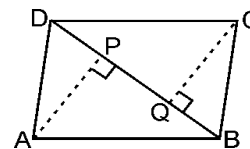
- (i) In  $\triangle APD$  and  $\triangle CQB$ , we have
  - $AD = BC$  [Opposite sides of a || gm]
  - $DP = BQ$  [Given]
  - $\angle ADP = \angle CBQ$  [Alternate angles]
  - $\therefore \triangle APD \cong \triangle CQB$  [SAS congruence]
- (ii)  $\therefore AP = CQ$  [CPCT]
- (iii) In  $\triangle AQB$  and  $\triangle CPD$ , we have
  - $AB = CD$  [Opposite sides of a || gm]
  - $DP = BQ$  [Given]
  - $\angle ABQ = \angle CDP$  [Alternate angles]
  - $\therefore \triangle AQB \cong \triangle CPD$  [SAS congruence]
- (iv)  $\therefore AQ = CP$  [CPCT]
- (v) Since in APCQ, opposite sides are equal, therefore it is a parallelogram. **Proved.**

**Q.10.** ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig.). Show that



- (i)  $\triangle APB \cong \triangle CQD$
- (ii)  $AP = CQ$

**Sol. Given :** ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on BD.

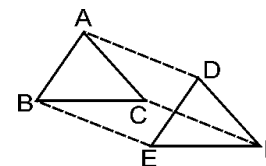


- To Prove :**
- (i)  $\triangle APB \cong \triangle CQD$
  - (ii)  $AP = CQ$

**Proof :**

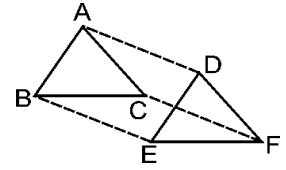
- (i) In  $\triangle APB$  and  $\triangle CQD$ , we have
  - $\angle ABP = \angle CDQ$  [Alternate angles]
  - $AB = CD$  [Opposite sides of a parallelogram]
  - $\angle APB = \angle CQD$  [Each =  $90^\circ$ ]
  - $\therefore \triangle APB \cong \triangle CQD$  [ASA congruence]
- (ii) So,  $AP = CQ$  [CPCT] **Proved.**

**Q.11.** In  $\triangle ABC$  and  $\triangle DEF$ ,  $AB = DE$ ,  $AB \parallel DE$ ,  $BC = EF$  and  $BC \parallel EF$ . Vertices A, B and C are joined to vertices D, E and F respectively (see Fig.). Show that



- (i) quadrilateral ABED is a parallelogram
- (ii) quadrilateral BEFC is a parallelogram
- (iii)  $AD \parallel CF$  and  $AD = CF$
- (iv) quadrilateral ACFD is a parallelogram
- (v)  $AC = DF$
- (vi)  $\triangle ABC \cong \triangle DEF$

**Sol. Given :** In  $\triangle ABC$  and  $\triangle DEF$ ,  $AB = DE$ ,  $AB \parallel DE$ ,  $BC = EF$  and  $BC \parallel EF$ . Vertices  $A, B$  and  $C$  are joined to vertices  $D, E$  and  $F$ .



- To Prove :**
- (i)  $ABED$  is a parallelogram
  - (ii)  $BEFC$  is a parallelogram
  - (iii)  $AD \parallel CF$  and  $AD = CF$
  - (iv)  $ACFD$  is a parallelogram
  - (v)  $AC = DF$
  - (vi)  $\triangle ABC \cong \triangle DEF$

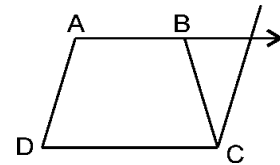
**Proof :**

- (i) In quadrilateral  $ABED$ , we have  
 $AB = DE$  and  $AB \parallel DE$ . [Given]  
 $\Rightarrow ABED$  is a parallelogram.  
 [One pair of opposite sides is parallel and equal]
- (ii) In quadrilateral  $BEFC$ , we have  
 $BC = EF$  and  $BC \parallel EF$  [Given]  
 $\Rightarrow BEFC$  is a parallelogram.  
 [One pair of opposite sides is parallel and equal]
- (iii)  $BE = CF$  and  $BE \parallel CF$  [BEFC is parallelogram]  
 $AD = BE$  and  $AD \parallel BE$  [ABED is a parallelogram]  
 $\Rightarrow AD = CF$  and  $AD \parallel CF$
- (iv)  $ACFD$  is a parallelogram.  
 [One pair of opposite sides is parallel and equal]
- (v)  $AC = DF$  [Opposite sides of parallelogram  $ACFD$ ]
- (vi) In  $\triangle ABC$  and  $\triangle DEF$ , we have  
 $AB = DE$  [Given]  
 $BC = EF$  [Given]  
 $AC = DF$  [Proved above]  
 $\therefore \triangle ABC \cong \triangle DEF$  [SSS axiom] **Proved.**

**Q.12.**  $ABCD$  is a trapezium in which  $AB \parallel CD$  and  $AD = BC$  (see Fig.).

Show that

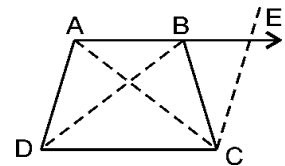
- (i)  $\angle A = \angle B$
- (ii)  $\angle C = \angle D$
- (iii)  $\triangle ABC \cong \triangle BAD$
- (iv) diagonal  $AC =$  diagonal  $BD$



**Sol. Given :** In trapezium  $ABCD$ ,  $AB \parallel CD$  and  $AD = BC$ .

- To Prove :**
- (i)  $\angle A = \angle B$
  - (ii)  $\angle C = \angle D$
  - (iii)  $\triangle ABC \cong \triangle BAD$
  - (iv) diagonal  $AC =$  diagonal  $BD$

**Constructions :** Join  $AC$  and  $BD$ . Extend  $AB$  and draw a line through  $C$  parallel to  $DA$  meeting  $AB$  produced at  $E$ .

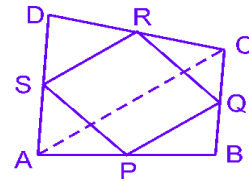


**Proof :**

- (i) Since  $AB \parallel DC$   
 $\Rightarrow AE \parallel DC$  ... (i)  
and  $AD \parallel CE$  ... (ii) [Construction]  
 $\Rightarrow ADCE$  is a parallelogram [Opposite pairs of sides are parallel]  
 $\angle A + \angle E = 180^\circ$  ... (iii) [Consecutive interior angles]  
 $\angle B + \angle CBE = 180^\circ$  ... (iv) [Linear pair]  
 $AD = CE$  ... (v) [Opposite sides of a  $\parallel^m$ ]  
 $AD = BC$  ... (vi) [Given]  
 $\Rightarrow BC = CE$  [From (v) and (vi)]  
 $\Rightarrow \angle E = \angle CBE$  ... (vii) [Angles opposite to equal sides]  
 $\therefore \angle B + \angle E = 180^\circ$  ... (viii) [From (iv) and (vii)]  
Now from (iii) and (viii) we have  
 $\angle A + \angle E = \angle B + \angle E$   
 $\Rightarrow \angle A = \angle B$  **Proved.**
- (ii)  $\left. \begin{array}{l} \angle A + \angle D = 180^\circ \\ \angle B + \angle C = 180^\circ \end{array} \right\}$  [Consecutive interior angles]  
 $\Rightarrow \angle A + \angle D = \angle B + \angle C$  [ $\because \angle A = \angle B$ ]  
 $\Rightarrow \angle D = \angle C$   
Or  $\angle C = \angle D$  **Proved.**
- (iii) In  $\triangle ABC$  and  $\triangle BAD$ , we have  
 $AD = BC$  [Given]  
 $\angle A = \angle B$  [Proved]  
 $AB = CD$  [Common]  
 $\therefore \triangle ABC \cong \triangle BAD$  [ASA congruence]
- (iv) diagonal  $AC =$  diagonal  $BD$  [CPCT] **Proved.**

### EXERCISE 8.2

**Q.1.** *ABCD* is a quadrilateral in which *P*, *Q*, *R* and *S* are mid-points of the sides *AB*, *BC*, *CD* and *DA* respectively. (see Fig.). *AC* is a diagonal. Show that :

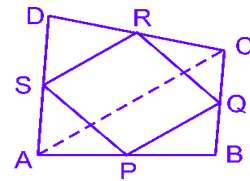


(i)  $SR \parallel AC$  and  $SR = \frac{1}{2}AC$

(ii)  $PQ = SR$

(iii) *PQRS* is a parallelogram.

**Given :** *ABCD* is a quadrilateral in which *P*, *Q*, *R* and *S* are mid-points of *AB*, *BC*, *CD* and *DA*. *AC* is a diagonal.



**To Prove :** (i)  $SR \parallel AC$  and  $SR = \frac{1}{2}AC$

(ii)  $PQ = SR$

(iii) *PQRS* is a parallelogram

**Proof :** (i) In  $\triangle ABC$ , *P* is the mid-point of *AB* and *Q* is the mid-point of *BC*.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC \quad \dots(1)$$

[Mid-point theorem]

In  $\triangle ADC$ , *R* is the mid-point of *CD* and *S* is the mid-point of *AD*

$$\therefore SR \parallel AC \text{ and } SR = \frac{1}{2}AC \quad \dots(2)$$

[Mid-point theorem]

(ii) From (1) and (2), we get

$$PQ \parallel SR \text{ and } PQ = SR$$

(iii) Now in quadrilateral *PQRS*, its one pair of opposite sides *PQ* and *SR* is equal and parallel.

$\therefore$  *PQRS* is a parallelogram. **Proved.**



**Q.2.** *ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.*

**Sol. Given :** ABCD is a rhombus in which P, Q, R and S are mid points of sides AB, BC, CD and DA respectively :

**To Prove :** PQRS is a rectangle.

**Construction :** Join AC, PR and SQ.

**Proof :** In  $\triangle ABC$

P is mid point of AB [Given]

Q is mid point of BC [Given]

$\Rightarrow PQ \parallel AC$  and  $PQ = \frac{1}{2} AC$  ... (i) [Mid point theorem]

Similarly, in  $\triangle DAC$ ,

$SR \parallel AC$  and  $SR = \frac{1}{2} AC$  ... (ii)

From (i) and (ii), we have  $PQ \parallel SR$  and  $PQ = SR$

$\Rightarrow$  PQRS is a parallelogram

[One pair of opposite sides is parallel and equal]

Since ABQS is a parallelogram

$\Rightarrow AB = SQ$  [Opposite sides of a  $\parallel$  gm]

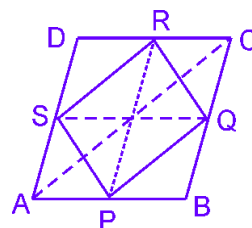
Similarly, since PBCR is a parallelogram.

$\Rightarrow BC = PR$

Thus,  $SQ = PR$  [AB = BC]

Since SQ and PR are diagonals of parallelogram PQRS, which are equal.

$\Rightarrow$  PQRS is a rectangle. **Proved.**

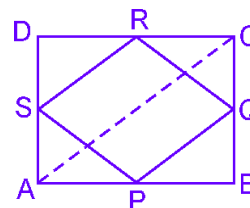


**Q.3.** *ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.*

**Sol. Given :** A rectangle ABCD in which P, Q, R, S are the mid-points of AB, BC, CD and DA respectively, PQ, QR, RS and SP are joined.

**To Prove :** PQRS is a rhombus.

**Construction :** Join AC



**Proof :** In  $\triangle ABC$ , P and Q are the mid-points of the sides AB and BC.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \dots(i) \quad [\text{Mid point theorem}]$$

Similarly, in  $\triangle ADC$ ,

$$SR \parallel AC \text{ and } SR = \frac{1}{2} AC \quad \dots(ii)$$

From (i) and (ii), we get

$$PQ \parallel SR \text{ and } PQ = SR \quad \dots(iii)$$

Now in quadrilateral PQRS, its one pair of opposite sides PQ and SR is parallel and equal [From (iii)]

$\therefore$  PQRS is a parallelogram.

$$\text{Now } AD = BC \quad \dots(iv)$$

[Opposite sides of a rectangle ABCD]

$$\therefore \frac{1}{2} AD = \frac{1}{2} BC$$

$$\Rightarrow AS = BQ$$

In  $\triangle APS$  and  $\triangle BPQ$

$$AP = BP$$

$$AS = BQ$$

$$\angle PAS = \angle PBQ$$

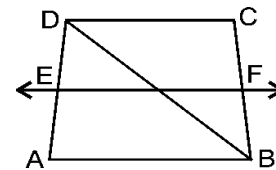
$$\triangle APS \cong \triangle BPQ$$

$$\therefore PS = PQ \quad \dots(v)$$

From (iii) and (v), we have

PQRS is a rhombus **Proved.**

**Q.4.** ABCD is a trapezium in which  $AB \parallel DC$ , BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig.). Show that F is the mid-point of BC.



**Sol. Given :** A trapezium ABCD with  $AB \parallel DC$ , E is the mid-point of AD and  $EF \parallel AB$ .

**To Prove :** F is the mid-point of BC.

**Proof :**  $AB \parallel DC$  and  $EF \parallel AB$

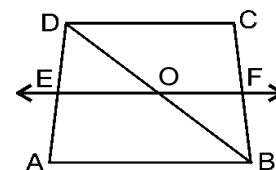
$\Rightarrow$  AB, EF and DC are parallel.

Intercepts made by parallel lines AB, EF and DC on transversal AD are equal.

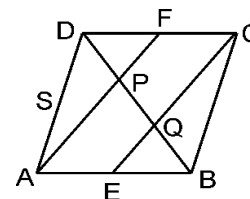
$\therefore$  Intercepts made by those parallel lines on transversal BC are also equal.

i.e.,  $BF = FC$

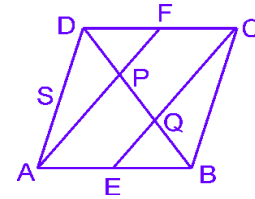
$\Rightarrow$  F is the mid-point of BC.



**Q.5.** In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig.). Show that the line segments AF and EC trisect the diagonal BD.



**Sol. Given :** A parallelogram ABCD, in which E and F are mid-points of sides AB and DC respectively.



**To Prove :**  $DP = PQ = QB$

**Proof :** Since E and F are mid-points of AB and DC respectively.

$$\Rightarrow AE = \frac{1}{2}AB \text{ and } CF = \frac{1}{2}DC \quad \dots(i)$$

$$\text{But, } AB = DC \text{ and } AB \parallel DC \quad \dots(ii)$$

[Opposite sides of a parallelogram]

$$\therefore AE = CF \text{ and } AE \parallel CF.$$

$\Rightarrow$  AECF is a parallelogram.

[One pair of opposite sides is parallel and equal]

In  $\triangle BAP$ ,

E is the mid-point of AB

$EQ \parallel AP$

$\Rightarrow$  Q is mid-point of PB [Converse of mid-point theorem]

$$\Rightarrow PQ = QB \quad \dots(iii)$$

Similarly, in  $\triangle DQC$ ,

P is the mid-point of DQ

$$DP = PQ \quad \dots(iv)$$

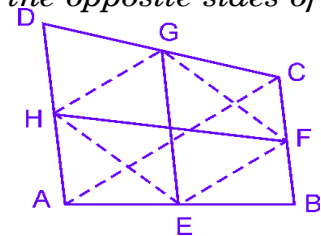
From (iii) and (iv), we have

$$DP = PQ = QB$$

or line segments AF and EC trisect the diagonal BD. **Proved.**

**Q.6.** Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

**Sol. Given :** ABCD is a quadrilateral in which EG and FH are the line segments joining the mid-points of opposite sides.



**To Prove :** EG and FH bisect each other.

**Construction :** Join EF, FG, GH, HE and AC.

**Proof :** In  $\triangle ABC$ , E and F are mid-points of AB and BC respectively.

$$\therefore EF = \frac{1}{2}AC \text{ and } EF \parallel AC \quad \dots(i)$$

In  $\triangle ADC$ , H and G are mid-points of AD and CD respectively.

$$\therefore HG = \frac{1}{2}AC \text{ and } HG \parallel AC \quad \dots(ii)$$

From (i) and (ii), we get

$$EF = HG \text{ and } EF \parallel HG$$

$\therefore$  EFGH is a parallelogram.

[ $\because$  a quadrilateral is a parallelogram if its one pair of opposite sides is equal and parallel]

Now, EG and FH are diagonals of the parallelogram EFGH.

$\therefore$  EG and FH bisect each other.

[Diagonal of a parallelogram bisect each other] **Proved.**

- Q.7.** *ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that*
- (i) *D is the mid-point of AC.*
  - (ii) *MD ⊥ AC*
  - (iii) *CM = MA =  $\frac{1}{2}$ AB*

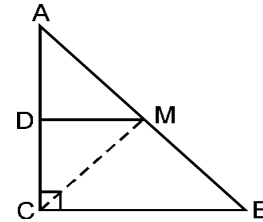
**Sol. Given :** A triangle ABC, in which  $\angle C = 90^\circ$  and M is the mid-point of AB and  $BC \parallel DM$ .

**To Prove :** (i) D is the mid-point of AC

[Given]

(ii)  $DM \perp BC$

(iii)  $CM = MA = \frac{1}{2}AB$



**Construction :** Join CM.

**Proof :** (i) In  $\triangle ABC$ ,

M is the mid-point of AB.

[Given]

$BC \parallel DM$

[Given]

D is the mid-point of AC

[Converse of mid-point theorem] **Proved.**

(ii)  $\angle ADM = \angle ACB$  [ $\because$  Corresponding angles]

But  $\angle ACB = 90^\circ$

[Given]

$\therefore \angle ADM = 90^\circ$

But  $\angle ADM + \angle CDM = 180^\circ$

[Linear pair]

$\therefore \angle CDM = 90^\circ$

Hence,  $MD \perp AC$  **Proved.**

(iii)  $AD = DC$  ...(1) [ $\because$  D is the mid-point of AC]

Now, in  $\triangle ADM$  and  $\triangle CMD$ , we have

$\angle ADM = \angle CDM$  [Each =  $90^\circ$ ]

$AD = DC$  [From (1)]

$DM = DM$  [Common]

$\therefore \triangle ADM \cong \triangle CMD$  [SAS congruence]

$\Rightarrow CM = MA$  ...(2) [CPCT]

Since M is mid-point of AB,

$\therefore MA = \frac{1}{2}AB$  ...(3)

Hence,  $CM = MA = \frac{1}{2}AB$  **Proved.** [From (2) and (3)]