CHAPTER - 10
CIRCLES

EXERCISE 10.1

Q.1. *Fill in the blanks :*

(i) The centre of a circle lies in __________ of the circle. (exterior/ interior)

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in __________ of the circle. (exterior/interior)

(iii) The longest chord of a circle is a __________ of the circle.

(iv) An arc is a __________ when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and __________ of the circle.

(vi) A circle divides the plane, on which it lies in __________ parts.

Sol. (i) interior (ii) exterior (iii) diameter (iv) semicircle (v) the chord (vi) three

Q.2. *Write True or False: Give reasons for your answers.*

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only finite number of equal chords.

(iii) If a circle is divided into three equal arcs, each is a major arc.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

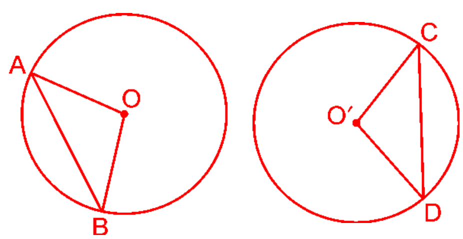
(vi) A circle is a plane figure.

Sol. (i) True (ii) False (iii) False (iv) True (v) False (vi) True

EXERCISE 10.2

Q.1. *Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.*

Sol. Given : Two congruent circles with centres O and O′. AB and CD are equal chords of the circles with centres O and O′ respectively.



To Prove : ∠AOB = ∠COD

Proof : In triangles AOB and COD,

AB = CD [Given]

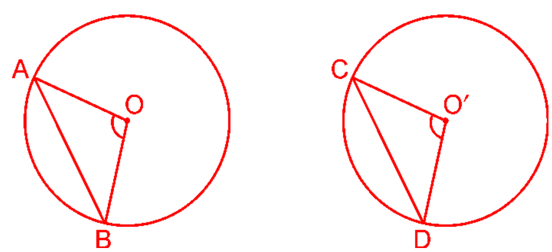
AO = CO′, BO = DO′ } [Radii of congruent circle]

⇒ ΔAOB ≅ ΔCO′D [SSS axiom]

⇒ ∠AOB ≅ ∠CO′D **Proved.** [CPCT]

Q.2. *Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.*

Ans. Given : Two congruent circles with centres O and O′. AB and CD are chords of circles with centre O and O′ respectively such that ∠AOB = ∠CO′D



To Prove : AB = CD

Proof : In triangles AOB and CO′D,

AO = CO′, BO = DO′ } [Radii of congruent circle]

∠AOB = ∠CO′D [Given]

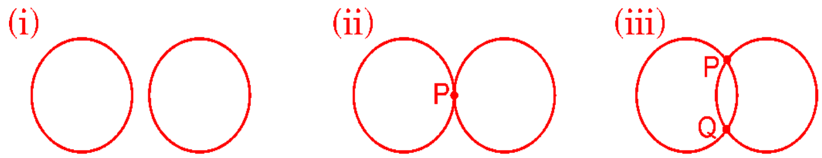
⇒ ΔAOB ≅ ΔCO′D [SAS axiom]

⇒ AB = CD **Proved.** [CPCT]

EXERCISE 10.3

Q.1. *Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?*

Ans.

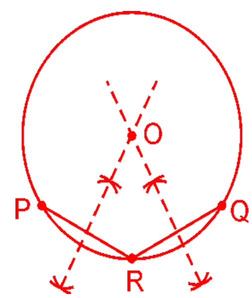


(i) 0 point (ii) 1 point (iii) 2 points

Maximum number of common points = 2 **Ans.**

Q.2. *Suppose you are given a circle. Give a construction to find its centre.*

Ans. **Steps of Construction :**



1. Take arc PQ of the given circle.
2. Take a point R on the arc PQ and draw chords PR and RQ.
3. Draw perpendicular bisectors of PR and RQ. These perpendicular bisectors intersect at point O.

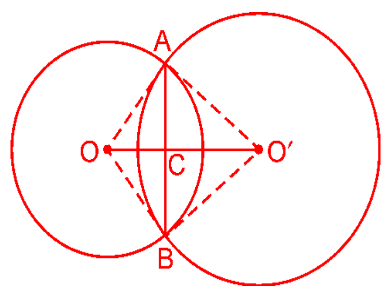
Hence, point O is the centre of the given circle.

Q.3. *If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.*

Ans. **Given :** AB is the common chord of two intersecting circles (O, r) and (O', r').

To Prove : Centres of both circles lie on the perpendicular bisector of chord AB, i.e., AB is bisected at right angle by OO'.

Construction : Join AO, BO, AO' and BO'.



Proof : In $\Delta AOO'$ and $\Delta BOO'$

$AO = OB$ (Radii of the circle (O, r))

$AO' = BO'$ (Radii of the circle (O', r'))

$OO' = OO'$ (Common)

$\therefore \Delta AOO' \cong \Delta BOO'$ (SSS congruency)

$\Rightarrow \angle AOO' = \angle BOO'$ (CPCT)

Now in ΔAOC and ΔBOC

$\angle AOC = \angle BOC$ $(\angle AOO' = \angle BOO')$

$AO = BO$ (Radii of the circle (O, r))

$OC = OC$ (Common)

$\therefore$ $\Delta AOC \cong \Delta BOC$ (SAS congruency)

$\Rightarrow$ $AC = BC$ and $\angle ACO = \angle BCO$ ...(i) (CPCT)

$\Rightarrow$ $\angle ACO + \angle BCO = 180°$ ..(ii) (Linear pair)

$\Rightarrow$ $\angle ACO = \angle BCO = 90°$ (From (i) and (ii))

Hence, OO′ lie on the perpendicular bisector of AB

EXERCISE 10.4

Q.1. *Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.*

Sol. In $\Delta AOO'$,

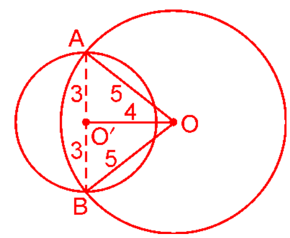
$AO^2 = 5^2 = 25$

$AO'^2 = 3^2 = 9$

$OO'^2 = 4^2 = 16$

$AO'^2 + OO'^2 = 9 + 16 = 25 = AO^2$

$\Rightarrow \angle AO'O = 90°$ [By converse of pythagoras theorem]



Similarly, $\angle BO'O = 90°$.

$\Rightarrow \angle AO'B = 90° + 90° = 180°$

$\Rightarrow$ AO′B is a straight line. whose mid-point is O.

$\Rightarrow AB = (3 + 3)$ cm $= 6$ cm **Ans.**

Q.2. *If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.*

Sol. Given : AB and CD are two equal chords of a circle which meet at E.

To prove : AE = CE and BE = DE

Construction : Draw $OM \perp AB$ and $ON \perp CD$ and join OE. **Proof :**

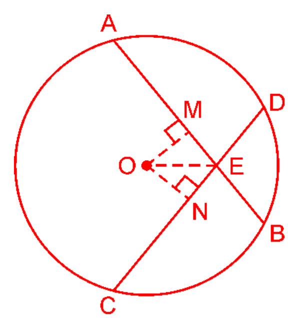
In ΔOME and ΔONE

OM = ON [Equal chords are equidistant]

OE = OE [Common]

$\angle OME = \angle ONE$ [Each equal to 90°]

$\therefore \Delta OME \cong \Delta ONE$ [RHS axiom]



$\Rightarrow$ EM = EN ...(i) [CPCT]

Now AB = CD [Given]

$\Rightarrow \frac{1}{2} AB = \frac{1}{2} CD$

$\Rightarrow$ AM = CN ..(ii) [Perpendicular from centre bisects the chord]

Adding (i) and (ii), we get

EM + AM = EN + CN

$\Rightarrow$ AE = CE ..(iii)

Now, AB = CD ..(iv)

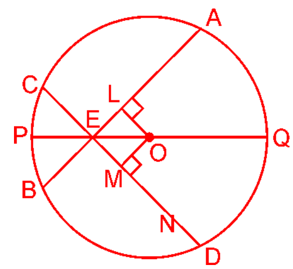
$\Rightarrow$ AB – AE = CD – AE [From (iii)]

$\Rightarrow$ BE = CD – CE **Proved.**

Q.3. *If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.*

Sol. Given : AB and CD are two equal chords of a circle which meet at E within the circle and a line PQ joining the point of intersection to the centre.

To Prove : $\angle AEQ = \angle DEQ$



Construction : Draw OL ⊥ AB and OM ⊥ CD.

Proof : In ΔOLE and ΔOME, we have

OL = OM [Equal chords are equidistant]

OE = OE [Common]

∠OLE = ∠OME [Each = 90°]

∴ ΔOLE ≅ ΔOME [RHS congruence]

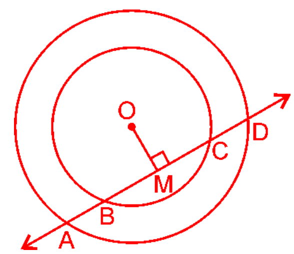
⇒ ∠LEO = ∠MEO [CPCT]

Q.4. *If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Fig.)*

Sol. **Given :** A line AD intersects two concentric circles at A, B, C and D, where O is the centre of these circles.

To prove : AB = CD

Construction : Draw OM ⊥ AD.



Proof : AD is the chord of larger circle.

∴ AM = DM ..(i) [OM bisects the chord]

BC is the chord of smaller circle

∴ BM = CM ..(ii) [OM bisects the chord]

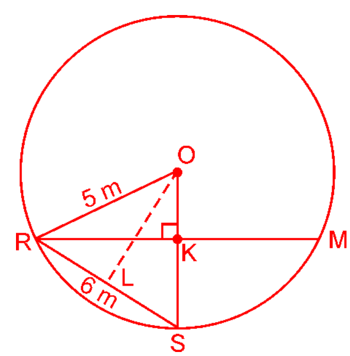
Subtracting (ii) from (i), we get

AM – BM = DM – CM

⇒ AB = CD **Proved.**

Q.5. *Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?*

Sol. Let Reshma, Salma and Mandip be represented by R, S and M respectively.



Draw OL ⊥ RS,

$OL^2 = OR^2 - RL^2$

$OL^2 = 5^2 - 3^2$ [RL = 3 m, because OL ⊥ RS]

$= 25 - 9 = 16$

$OL = \sqrt{16} = 4$

Now, area of triangle ORS $= \frac{1}{2} \times KR \times 05$

$= \frac{1}{2} \times KR \times 05$

Also, area of ΔORS $= \frac{1}{2} \times RS \times OL = \frac{1}{2} \times 6 \times 4 = 12\ m^2$

$\Rightarrow \frac{1}{2} \times KR \times 5 = 12$

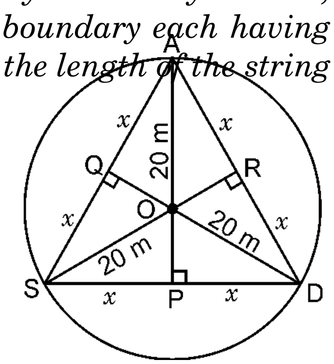
$\Rightarrow KR = \frac{12 \times 2}{5} = \frac{24}{5} = 4.8$ m

⇒ RM = 2KR

⇒ RM = 2 × 4.8 = 9.6 m

Hence, distance between Reshma and Mandip is 9.6 m **Ans.**

Q.6. *A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are siting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.*



Sol. Let Ankur, Syed and David be represented by A, S and D respectively.

Let PD = SP = SQ = QA = AR = RD = x

In ΔOPD,

$OP^2 = 400 - x^2$

$\Rightarrow \quad OP = \sqrt{400 - x^2}$

$\Rightarrow \quad AP = 2\sqrt{400 - x^2} + \sqrt{400 - x^2}$

[$\because$ centroid divides the median in the ratio 2 : 1]

$= 3\sqrt{400 - x^2}$

Now, in ΔAPD,

$PD^2 = AD^2 - DP^2$

$\Rightarrow \quad x^2 = (2x)^2 - \left(3\sqrt{400 - x^2}\right)^2$

$\Rightarrow \quad x^2 = 4x^2 - 9(400 - x^2)$

$\Rightarrow \quad x^2 = 4x^2 - 3600 + 9x^2$

$\Rightarrow \quad 12x^2 = 3600$

$\Rightarrow \quad x^2 = \frac{3600}{12} = 300$

$\Rightarrow \quad x = 10\sqrt{3}$

Now, SD = $2x = 2 \times 10\sqrt{3} = 20\sqrt{3}$

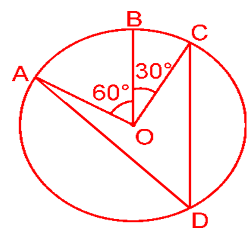
$\therefore$ ASD is an equilateral triangle.

$\Rightarrow$ SD = AS = AD = $20\sqrt{3}$

Hence, length of the string of each phone is $20\sqrt{3}$ m **Ans.**

EXERCISE 10.5

Q.1. *In the figure, A, B and C are three points on a circle with centre O such that ∠ BOC = 30° and ∠ AOB = 60°. If D is a point on the circle other than the arc ABC, find ∠ ADC.*



Sol. We have, ∠BOC = 30° and ∠AOB = 60°

∠AOC = ∠AOB + ∠BOC = 60° + 30° = 90°

We know that angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc on the remaining part of the circle.

∴ 2∠ADC = ∠AOC

$\Rightarrow$ $\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2} \times 90°$ $\Rightarrow$ ∠ADC = 45° **Ans.**

Q.2. *A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.*

Sol. We have, OA = OB = AB

Therefore, ΔOAB is a equilateral triangle.

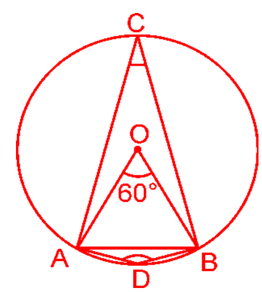
$\Rightarrow$ ∠AOB = 60°

We know that angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc on the remaining part of the circle.

∴ ∠AOB = 2∠ACB

$\Rightarrow$ $\angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 60°$

$\Rightarrow$ ∠ACB = 30°



Also, $\angle ADB = \frac{1}{2}$ reflex ∠AOB

$= \frac{1}{2}(360° - 60°) = \frac{1}{2} \times 300° = 150°$

Hence, angle subtended by the chord at a point on the minor arc is 150° and at a point on the major arc is 30° **Ans.**

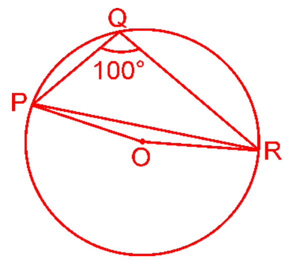
Q.3. *In the figure, ∠PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠OPR.*

Sol. Reflex angle POR = 2∠PQR

= 2 × 100° = 200°

Now, angle POR = 360° – 200 = 160°

Also,



PO = OR [Radii of a circle]

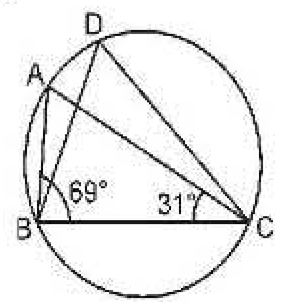
∠OPR = ∠ORP [Opposite angles of isosceles triangle]

In ΔOPR, ∠POR = 160°

∴ ∠OPR = ∠ORP = 10°

[Angle sum property of a triangle]. **Ans.**

Q.4. *In the figure, ∠ ABC = 69°, ∠ ACB = 31°, find ∠ BDC.*



Sol. In ΔABC, we have

∠ABC + ∠ACB + ∠BAC = 180°

[Angle sum property of a triangle]

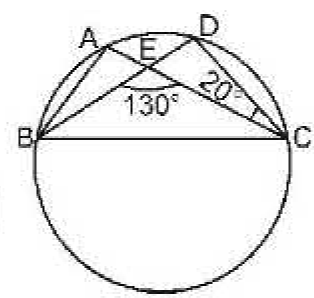
⇒ 69° + 31° + ∠BAC = 180°

⇒ ∠BAC = 180° – 100° = 80°

Also, ∠BAC= ∠BDC [Angles in the same segment]

⇒ ∠BDC = 80° **Ans.**

Q.5. *In the figrue, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠BEC = 130° and ∠ECD = 20°. Find ∠BAC.*



Sol. ∠BEC + ∠DEC = 180° [Linear pair]

⇒ 130° + ∠DEC = 180°

⇒ ∠DEC = 180° – 130° = 50°

Now, in ΔDEC,

⇒ ∠DEC + ∠DCE + ∠CDE = 180°

[Angle sum property of a triangle]

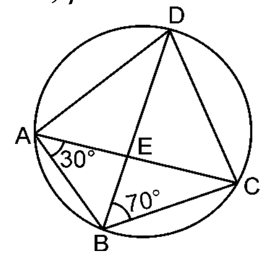
⇒ 50° + 20° + ∠CDE = 180°

⇒ ∠CDE = 180° – 70° = 110°

Also, ∠CDE = ∠BAC [Angles in same segment]

⇒ ∠BAC = 110° **Ans.**

Q.6. *ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70°$, $\angle BAC = 30°$, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$.*



Sol. $\angle CAD = \angle DBC = 70°$ [Angles in the same segment]

Therefore, $\angle DAB = \angle CAD + \angle BAC$

$= 70° + 30° = 100°$

But, $\angle DAB + \angle BCD = 180°$

[Opposite angles of a cyclic quadrilateral]

So, $\angle BCD = 180° - 100° = 80°$

Now, we have AB = BC

Therefore, $\angle BCA = 30°$ [Opposite angles of an isosceles triangle]

Again, $\angle DAB + \angle BCD = 180°$

[Opposite angles of a cyclic quadrilateral]

$\Rightarrow 100° + \angle BCA + \angle ECD = 180°$ [$\because \angle BCD = \angle BCA + \angle ECD$]

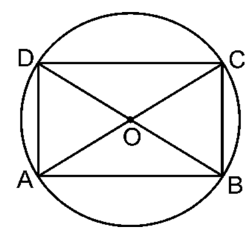
$\Rightarrow 100° + 30° + \angle ECD = 180°$

$\Rightarrow 130° + \angle ECD = 180°$

$\Rightarrow \angle ECD = 180° - 130° = 50°$

Hence, $\angle BCD = 80°$ and $\angle ECD = 50°$ **Ans.**

Q.7. *If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.*



Sol. **Given :** ABCD is a cyclic quadrilateral, whose diagonals AC and BD are diameter of the circle passing through A, B, C and D.

To Prove : ABCD is a rectangle.

Proof : In ΔAOD and ΔCOB

AO = CO [Radii of a circle]

OD = OB [Radii of a circle]

$\angle AOD = \angle COB$ [Vertically opposite angles]

$\therefore$ $\Delta AOD \cong \Delta COB$ [SAS axiom]

$\therefore$ $\angle OAD = \angle OCB$ [CPCT]

But these are alternate interior angles made by the transversal AC, intersecting AD and BC.

$\therefore$ AD || BC

Similarly, AB || CD.

Hence, quadrilateral ABCD is a parallelogram.

Also, $\angle ABC = \angle ADC$ ..(i) [Opposite angles of a ||gm are equal]

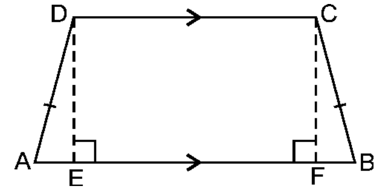
And, $\angle ABC + \angle ADC = 180°$ ...(ii)

[Sum of opposite angles of a cyclic quadrilateral is 180°]

$\Rightarrow \angle ABC = \angle ADC = 90°$ [From (i) and (ii)]

$\therefore$ ABCD is a rectangle. [A ||gm one of whose angles is 90° is a rectangle] **Proved.**

Q.8. *If the non-parallel sides of a trapezium are equal, prove that it is cyclic.*



Sol. **Given :** A trapezium ABCD in which AB || CD and AD = BC.

To Prove : ABCD is a cyclic trapezium.

Construction : Draw DE ⊥ AB and CF ⊥ AB.

Proof : In ΔDEA and ΔCFB, we have

AD = BC [Given]

$\angle DEA = \angle CFB = 90°$ [DE ⊥ AB and CF ⊥ AB]

$DE = CF$

[Distance between parallel lines remains constant]

$\therefore \quad \Delta DEA \cong \Delta CFB$ [RHS axiom]

$\Rightarrow \quad \angle A = \angle B$ ...(i) [CPCT]

and, $\quad \angle ADE = \angle BCF$ ..(ii) [CPCT]

Since, $\quad \angle ADE = \angle BCF$ [From (ii)]

$\Rightarrow \quad \angle ADE + 90° = \angle BCF + 90°$

$\Rightarrow \quad \angle ADE + \angle CDE = \angle BCF + \angle DCF$

$\Rightarrow \quad \angle D = \angle C$ ..(iii)

[$\angle ADE + \angle CDE = \angle D, \angle BCF + \angle DCF = \angle C$]

$\therefore \angle A = \angle B$ and $\angle C = \angle D$ [From (i) and (iii)] (iv)

$\angle A + \angle B + \angle C + \angle D = 360°$ [Sum of the angles of a quadrilateral is 360°]

$\Rightarrow 2(\angle B + \angle D) = 360°$ [Using (iv)]

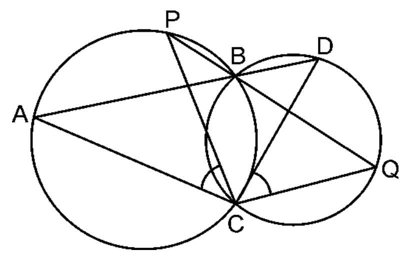
$\Rightarrow \angle B + \angle D = 180°$

$\Rightarrow$ Sum of a pair of opposite angles of quadrilateral ABCD is 180°.

$\Rightarrow$ ABCD is a cyclic trapezium **Proved.**

Q.9. *Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig.). Prove that ∠ ACP = ∠QCD.*

Sol. Given : Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively.



To Prove : $\angle ACP = \angle QCD$.

Proof : $\angle ACP = \angle ABP$ ...(i)

[Angles in the same segment]

$\angle QCD = \angle QBD$ ..(ii)

[Angles in the same segment]

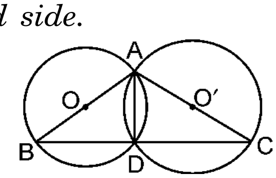
But, $\quad \angle ABP = \angle QBD$ ..(iii) [Vertically opposite angles]

By (i), (ii) and (ii) we get

$\angle ACP = \angle QCD$ **Proved.**

Q.10. *If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.*

Sol. Given : Sides AB and AC of a triangle ABC are diameters of two circles which intersect at D.



To Prove : D lies on BC.

Proof : Join AD

$\angle ADB = 90°$ ...(i) [Angle in a semicircle]

Also, $\quad \angle ADC = 90°$ ..(ii)

Adding (i) and (ii), we get

$\angle ADB + \angle ADC = 90° + 90°$

$\Rightarrow \quad \angle ADB + \angle ADC = 180°$

$\Rightarrow$ BDC is a straight line.

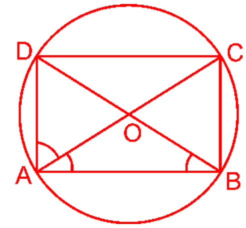
$\therefore$ D lies on BC

Hence, point of intersection of circles lie on the third side BC. **Proved.**

Q.11. *ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠CAD = ∠CBD.*

Sol. Given : ABC and ADC are two right triangles with common hypotenuse AC.

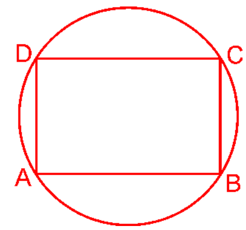
To Prove : $\angle CAD = \angle CBD$



Proof : Let O be the mid-point of AC.
Then OA = OB = OC = OD
Mid point of the hypotenuse of a right triangle is equidistant from its vertices with O as centre and radius equal to OA, draw a circle to pass through A, B, C and D.
We know that angles in the same segment of a circle are equal.
Since, $\angle CAD$ and $\angle CBD$ are angles of the same segment.
Therefore, $\angle CAD = \angle CBD$. **Proved.**

Q.12. *Prove that a cyclic parallelogram is a rectangle.*

Sol. **Given :** ABCD is a cyclic parallelogram.

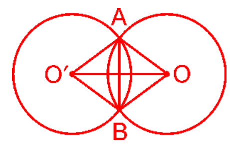


To prove : ABCD is a rectangle.
Proof : $\angle ABC = \angle ADC$ ...(i)
[Opposite angles of a ||gm are equal]
But, $\angle ABC + \angle ADC = 180°$ ... (ii)
[Sum of opposite angles of a cyclic quadrilateral is 180°]
$\Rightarrow \angle ABC = \angle ADC = 90°$ [From (i) and (ii)]
$\therefore$ ABCD is a rectangle
[A ||gm one of whose angles is 90° is a rectangle]
Hence, a cyclic parallelogram is a rectangle. **Proved.**

EXERCISE 10.6 (Optional)

Q.1. *Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.*

Sol. **Given :** Two intersecting circles, in which OO′ is the line of centres and A and B are two points of intersection.



To prove : ∠OAO′ = ∠OBO′

Construction : Join AO, BO, AO′ and BO′.

Proof : In ΔAOO′ and ΔBOO′, we have

AO = BO [Radii of the same circle]

AO′ = BO′ [Radii of the same circle]

OO′ = OO′ [Common]

∴ ΔAOO′ ≅ ΔBOO′ [SSS axiom]

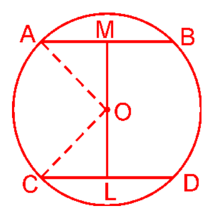
⇒ ∠OAO′ = ∠OBO′ [CPCT]

Hence, the line of centres of two intersecting circles subtends equal angles at the two points of intersection. **Proved.**

Q.2. *Two chords AB and CD of lengths* 5 *cm and* 11 *cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is* 6 *cm, find the radius of the circle.*

Sol. Let O be the centre of the circle and let its radius be r cm.

Draw OM ⊥ AB and OL ⊥ CD.



Then, $AM = \frac{1}{2}AB = \frac{5}{2}$ cm

and, $CL = \frac{1}{2}CD = \frac{11}{2}$ cm

Since, AB || CD, it follows that the points O, L, M are collinear and therefore, LM = 6 cm.

Let OL = x cm. Then OM = $(6 - x)$ cm

Join OA and OC. Then OA = OC = r cm.

Now, from right-angled ΔOMA and ΔOLC, we have

$OA^2 = OM^2 + AM^2$ and $OC^2 = OL^2 + CL^2$ [By Pythagoras Theorem]

$\Rightarrow r^2 = (6 - x)^2 + \left(\frac{5}{2}\right)^2$ ..(i) and $r^2 = x^2 + \left(\frac{11}{2}\right)^2$ ... (ii)

$$\Rightarrow (6-x)^2 + \left(\frac{5}{2}\right)^2 = x^2 + \left(\frac{11}{2}\right)^2 \quad \text{[From (i) and (ii)]}$$

$$\Rightarrow 36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$$

$$\Rightarrow -12x = \frac{121}{4} - \frac{25}{4} - 36$$

$$\Rightarrow -12x = \frac{96}{4} - 36$$

$$\Rightarrow -12x = 24 - 36$$

$$\Rightarrow -12x = -12$$

$$\Rightarrow \quad x = 1$$

Substituting $x = 1$ in (i), we get

$$r^2 = (6-x)^2 + \left(\frac{5}{2}\right)^2$$

$$\Rightarrow r^2 = (6-1)^2 + \left(\frac{5}{2}\right)^2$$

$$\Rightarrow r^2 = (5)^2 + \left(\frac{5}{2}\right)^2 = 25 + \frac{25}{4}$$

$$\Rightarrow r^2 = \frac{125}{4}$$

$$\Rightarrow r = \frac{5\sqrt{5}}{2}$$

Hence, radius $r = \frac{5\sqrt{5}}{2}$ cm. **Ans.**

Q.3. *The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?*

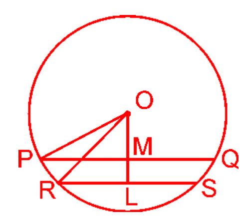
Sol. Let PQ and RS be two parallel chords of a circle with centre O.

We have, PQ = 8 cm and RS = 6 cm.

Draw perpendicular bisector OL of RS which meets PQ in M. Since, PQ || RS, therefore, OM is also perpendicular bisector of PQ.

Also, OL = 4 cm and $RL = \frac{1}{2}RS \Rightarrow RL = 3$ cm

and $PM = \frac{1}{2}PQ \Rightarrow PM = 4$ cm



In ΔORL, we have

$OR^2 = RL^2 + OL^2$ [Pythagoras theorem]

$\Rightarrow OR^2 = 3^2 + 4^2 = 9 + 16$

$\Rightarrow OR^2 = 25 \Rightarrow OR = \sqrt{25}$

$\Rightarrow OR = 5$ cm

$\therefore OR = OP$ [Radii of the circle]

$\Rightarrow OP = 5$ cm

Now, in ΔOPM

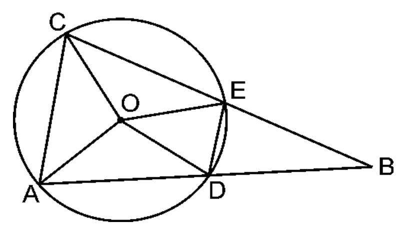
$OM^2 = OP^2 - PM^2$ [Pythagoras theorem]

$\Rightarrow OM^2 = 5^2 - 4^2 = 25 - 16 = 9$

$OM = \sqrt{9} = 3$ cm

Hence, the distance of the other chord from the centre is 3 cm. **Ans.**

Q.4. *Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.*



Sol. Given : Two equal chords AD and CE of a circle with centre O. When meet at B when produced.

To Prove : $\angle ABC = \frac{1}{2}(\angle AOC - \angle DOE)$

Proof : Let $\angle AOC = x$, $\angle DOE = y$, $\angle AOD = z$

$\angle EOC = z$ [Equal chords subtends equal angles at the centre]

$\therefore x + y + 2z = 36°$ [Angle at a point] .. (i)

$OA = OD \Rightarrow \angle OAD = \angle ODA$

$\therefore$ In DOAD, we have

$\angle OAD + \angle ODA + z = 180°$

$\Rightarrow 2\angle OAD = 180° - z$ [$\because \angle OAD = \angle OBA$]

$\Rightarrow \angle OAD = 90° - \frac{z}{2}$ ... (ii)

Similarly $\angle OCE = 90° - \frac{z}{2}$ ... (iii)

$\Rightarrow \angle ODB = \angle OAD + \angle ODA$ [Exterior angle property]

$\Rightarrow \angle OEB = 90° - \frac{z}{2} + z$ [From (ii)]

$\Rightarrow \angle ODB = 90° + \frac{z}{2}$ ... (iv)

Also, $\angle OEB = \angle OCE + \angle COE$ [Exterior angle property]

$\Rightarrow \angle OEB = 90° - \frac{z}{2} + z$ [From (iii)]

$\Rightarrow \angle OEB = 90° + \frac{z}{2}$ ... (v)

Also, $\angle OED = \angle ODE = 90° - \frac{y}{2}$ ... (vi)

O from (iv), (v) and (vi), we have

$$\angle BDE = \angle BED = 90° + \frac{z}{2} - \left(90° - \frac{y}{2}\right)$$

$\Rightarrow\ \angle BDE = \angle BED = \frac{y + z}{2}$

$\Rightarrow\ \angle BDE = \angle BED = y + z$ ... (vii)

$\therefore\ \angle BDE = 180° - (y + z)$

$\Rightarrow\ \angle ABC = 180° - (y + z)$ ... (viii)

Now, $\frac{y - z}{2} = \frac{360° - y - 2z - y}{2} = 180° - (y + z)$ ... (ix)

From (viii) and (ix), we have

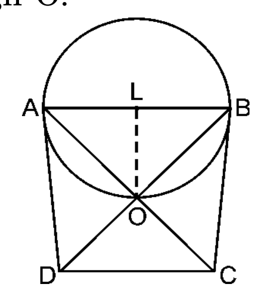
$\angle ABC = \frac{x - y}{2}$ **Proved.**

Q.5. *Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.*

Sol. **Given :** A rhombus ABCD whose diagonals intersect each other at O.

To prove : A circle with AB as diameter passes through O.

Proof : $\angle AOB = 90°$



[Diagonals of a rhombus bisect each other at 90°]

$\Rightarrow$ ΔAOB is a right triangle right angled at O.

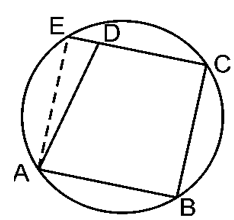
$\Rightarrow$ AB is the hypotenuse of A B right ΔAOB.

$\Rightarrow$ If we draw a circle with AB as diameter, then it will pass through O. because angle is a semicircle is 90° and $\angle AOB = 90°$ **Proved.**

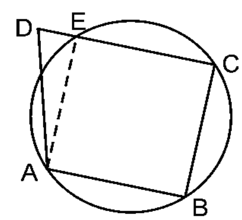
Q.6. *ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.*

Sol. **Given :** ABCD is a parallelogram.

To Prove : AE = AD.



(i)



(ii)

Construction : Draw a circle which passes through ABC and intersect CD (or CD produced) at E.

Proof : For fig (i)

$\angle AED + \angle ABC = 180°$

[Linear pair] ... (ii)

But $\angle ACD = \angle ADC = \angle ABC + \angle ADE$

$\Rightarrow\ \angle ABC + \angle ADE = 180°$ [From (ii)] ... (iii)

From (i) and (iii)

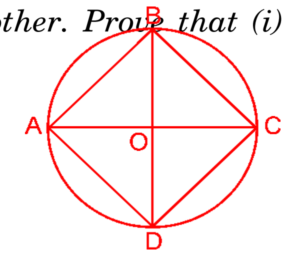
$\angle AED + \angle ABC = \angle ABC + \angle ADE$

$\Rightarrow\ \angle AED = \angle ADE$

$\Rightarrow\ \angle AD = \angle AE$ [Sides opposite to equal angles are equal]

Similarly we can prove for Fig (ii) **Proved.**

Q.7. *AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters, (ii) ABCD is rectangle.*



Sol. Given : A circle with chords AB and CD which bisect each other at O.

To Prove : (i) AC and BD are diameters

(ii) ABCD is a rectangle.

Proof : In ΔOAB and ΔOCD, we have

$OA = OC$ [Given]

$OB = OD$ [Given]

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\Rightarrow \Delta AOB \cong \angle COD$ [SAS congruence]

$\Rightarrow \angle ABO = \angle CDO$ and $\angle BAO = \angle BCO$ [CPCT]

$\Rightarrow AB \,||\, DC$ ... (i)

Similarly, we can prove $BC \,||\, AD$ ... (ii)

Hence, ABCD is a parallelogram.

But ABCD is a cyclic parallelogram.

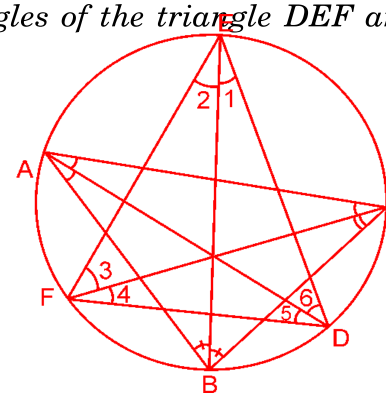
$\therefore$ ABCD is a rectangle. [Proved in Q. 12 of Ex. 10.5]

$\Rightarrow \angle ABC = 90°$ and $\angle BCD = 90°$

$\Rightarrow$ AC is a diameter and BD is a diameter

[Angle in a semicircle is 90°] **Proved.**

Q.8. *Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are* $90° - \frac{1}{2}\angle A$, $90° - \frac{1}{2}\angle B$ *and* $90° - \frac{1}{2}\angle C$.



Sol. Given : ΔABC and its circumcircle. AD, BE, CF are bisectors of $\angle A$, $\angle B$, $\angle C$ respectively.

Construction : Join DE, EF and FD.

Proof : We know that angles in the same segment are equal.

$\therefore \quad \angle 5 = \frac{\angle C}{2}$ and $\angle 6 = \frac{\angle B}{2}$ ..(i)

$\angle 1 = \frac{\angle A}{2}$ and $\angle 2 = \frac{\angle C}{2}$ ..(ii)

$\angle 4 = \frac{\angle A}{2}$ and $\angle 3 = \frac{\angle B}{2}$ ..(iii)

From (i), we have

$$\angle 5 + \angle 6 = \frac{\angle C}{2} + \frac{\angle B}{2}$$

$$\Rightarrow \quad \angle D = \frac{\angle C}{2} + \frac{\angle B}{2} \quad ...(iv)$$

[$\because \angle 5 + \angle 6 = \angle D$]

But $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle B + \angle C = 180° - \angle A$

$$\Rightarrow \qquad \frac{\angle B}{2} + \frac{\angle C}{2} = 90° - \frac{\angle A}{2}$$

$\therefore$ (iv) becomes,

$\angle D = 90° - \dfrac{\angle A}{2}$.

Similarly, from (ii) and (iii), we can prove that

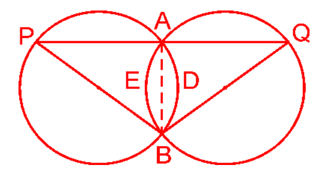
$\angle E = 90° - \dfrac{\angle B}{2}$ and $\angle F = 90° - \dfrac{\angle C}{2}$ **Proved.**

Q.9. *Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.*

Sol. Given : Two congruent circles which intersect at A and B. PAB is a line through A.

To Prove : BP = BQ.

Construction : Join AB.



Proof : AB is a common chord of both the circles.

But the circles are congruent —

$\Rightarrow$ arc ADB = arc AEB

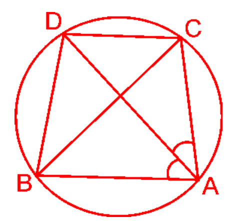
$\Rightarrow \quad \angle APB = \angle AQB$ Angles subtended

$\Rightarrow \quad BP = BQ$ [Sides opposite to equal angles are equal] **Proved.**

Q.10. *In any triangle ABC, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.*

Sol. Let angle bisector of $\angle A$ intersect circumcircle of ΔABC at D.

Join DC and DB.



$\angle BCD = \angle BAD$

[Angles in the same segment]

$\Rightarrow \angle BCD = \angle BAD \ \dfrac{1}{2} \ \angle A$

[AD is bisector of $\angle A$] ...(i)

Similarly $\angle DBC = \angle DAC \ \dfrac{1}{2} \ \angle A$ ... (ii)

From (i) and (ii) $\angle DBC = \angle BCD$

$\Rightarrow$ BD = DC [sides opposite to equal angles are equal]

$\Rightarrow$ D lies on the perpendicular bisector of BC.

Hence, angle bisector of $\angle A$ and perpendicular bisector of BC intersect on the circumcircle of ΔABC **Proved.**