

CHAPTER - 12

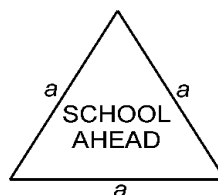
HERON'S FORMULA

12.1

Q.1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?

Sol. Each side of the triangle = a
Perimeter of the triangle = 3a

$$\therefore s = \frac{3a}{2}$$



$$\begin{aligned} \therefore \text{Area of the signal board (triangle)} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{s(s-a)(s-a)(s-a)} \quad [\because a = b = c] \\ &= (s-a)\sqrt{s(s-a)} = \left(\frac{3a}{2} - a\right) \sqrt{\frac{3a}{2} \left(\frac{3a}{2} - a\right)} \\ &= \frac{a}{2} \cdot \sqrt{\frac{3a^2}{4}} = \frac{a}{2} \cdot \frac{a}{2} \sqrt{3} = \frac{a^2}{4} \sqrt{3} \end{aligned}$$

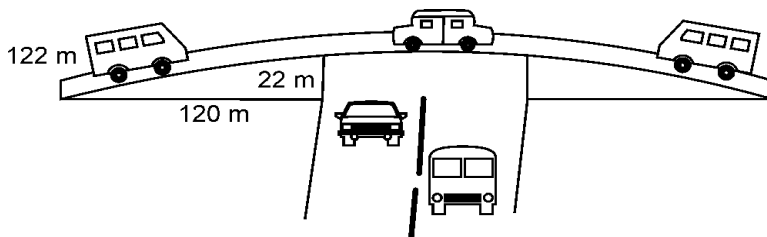
Hence, area of the signal board = $\frac{a^2}{4} \sqrt{3}$ sq units **Ans.**

Now, perimeter = 180 cm

$$\text{Each side of the triangle} = \frac{180}{3} \text{ cm} = 60 \text{ cm}$$

$$\text{Area of the triangle} = \frac{(60)^2}{4} \times \sqrt{3} \text{ cm}^2 = 900 \sqrt{3} \text{ cm}^2 \text{ **Ans.**}$$

Q.2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig.). The advertisements yield an earning of Rs 5000 per m² per year. A company hired one of its walls for 3 months. How much rent did it pay?



Sol. Here, we first find the area of the triangular side walls.

$$a = 122 \text{ m}, \quad b = 120 \text{ m} \text{ and } c = 22 \text{ m}$$

$$\therefore s = \frac{122 + 120 + 22}{2} \text{ m} = 132 \text{ m.}$$

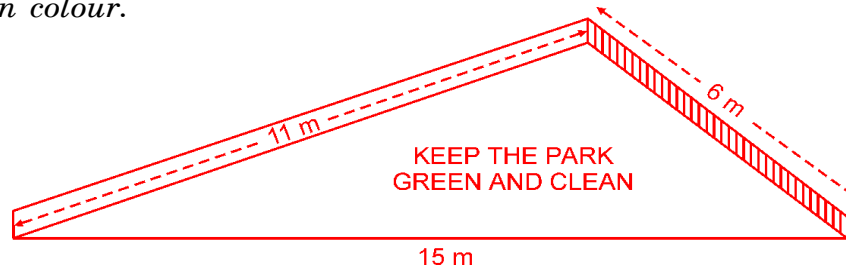
$$\begin{aligned} \text{Area of the triangular side wall} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{132(132-122)(132-120)(132-22)} \text{ m}^2 \\ &= \sqrt{132 \times 10 \times 12 \times 110} \text{ m}^2 = 1320 \text{ m}^2 \end{aligned}$$

Rent of 1 m² of the wall for 1 year = Rs 5000

$$\therefore \text{Rent of 1 m}^2 \text{ of the wall for 1 month} = \text{Rs } \frac{5000}{12}$$

$$\begin{aligned} \therefore \text{Rent of the complete wall (1320 m}^2\text{) for 3 months} \\ &= \text{Rs } \frac{5000}{12} \times 1320 \times 3 = \text{Rs } 16,50,000 \text{ Ans.} \end{aligned}$$

Q.3. *There is a slide in a park. One of its side walls has been painted in some colour with a message “KEEP THE PARK GREEN AND CLEAN” (see Fig.). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.*



Sol. Here $a = 15$ m, $b = 11$ m, $c = 6$ m

$$\therefore s = \frac{a+b+c}{2} = \frac{15+11+6}{2} \text{ m} = 16 \text{ m}$$

$$\begin{aligned} \text{Area of the triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{16(16-15)(16-11)(16-6)} \text{ m}^2 \\ &= \sqrt{16 \times 1 \times 5 \times 10} \text{ m}^2 = 20\sqrt{2} \text{ m}^2 \end{aligned}$$

Hence, the area painted in colour = $20\sqrt{2} \text{ m}^2$ Ans.

Q.4. *Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.*

Sol. Here $a = 18$ cm, $b = 10$ cm, $c = ?$

Perimeter of the triangle = 42 cm

$$\Rightarrow a + b + c = 42$$

$$\Rightarrow 18 + 10 + c = 42$$

$$\Rightarrow c = 42 - 28 = 14$$

$$\text{Now, } s = \frac{a+b+c}{2} = \frac{42}{2} \text{ cm} = 21 \text{ cm}$$

$$\begin{aligned} \text{Area of the triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{21(21-18)(21-10)(21-14)} \text{ cm}^2 \\ &= \sqrt{21 \times 3 \times 11 \times 7} \text{ cm}^2 = \sqrt{7 \times 3 \times 3 \times 11 \times 7} \text{ cm}^2 \\ &= 7 \times 3 \sqrt{11} \text{ cm}^2 = 21\sqrt{11} \text{ cm}^2 \text{ Ans.} \end{aligned}$$

Q.5. Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540 cm. Find its area.

Sol. Let the sides of the triangle be $12x$ cm, $17x$ cm and $25x$ cm.

Perimeter of the triangle = 540 cm

$$\therefore 12x + 17x + 25x = 540$$

$$\Rightarrow 54x = 540$$

$$\Rightarrow x = \frac{540}{54} = 10$$

\therefore Sides of the triangle are (12×10) cm, (17×10) cm and (25×10) cm i.e., 120 cm, 170 cm and 250 cm.

Now, suppose $a = 120$ cm, $b = 170$ cm, $c = 250$ cm,

$$\therefore s = \frac{a + b + c}{2} = \frac{540}{2} \text{ cm} = 270 \text{ cm}$$

$$\begin{aligned} \text{Area of the triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{270(270-120)(270-170)(270-250)} \text{ cm}^2 \\ &= \sqrt{270 \times 150 \times 100 \times 20} \text{ cm}^2 = \mathbf{9000 \text{ cm}^2 \text{ Ans.}} \end{aligned}$$

Q.6. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Sol. Here, $a = b = 12$ cm,

$$\text{Also, } a + b + c = 30 \quad \Rightarrow 12 + 12 + c = 30 \quad \Rightarrow c = 30 - 24 = 6$$

$$\therefore s = \frac{a + b + c}{2} = \frac{30}{2} \text{ cm} = 15 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of the triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{15(15-12)(15-12)(15-6)} \text{ cm}^2 \\ &= \sqrt{15 \times 3 \times 3 \times 9} \text{ cm}^2 = \mathbf{9\sqrt{15} \text{ cm}^2 \text{ Ans.}} \end{aligned}$$

EXERCISE 12.2

Q.1. A park, in the shape of a quadrilateral $ABCD$, has $\angle C = 90^\circ$, $AB = 9$ m, $BC = 12$ m, $CD = 5$ m and $AD = 8$ m. How much area does it occupy?

Sol. $ABCD$ is the park as shown in the figure.

Join BD .

In $\triangle DBC$, we have

$$DB^2 = BC^2 + CD^2 \quad [\text{Pythagoras theorem}]$$

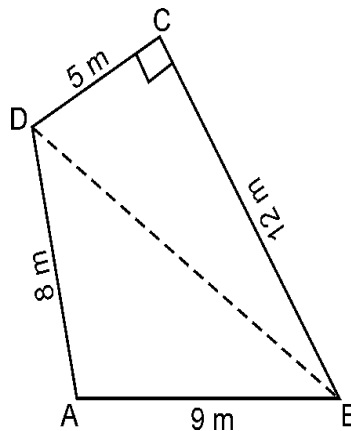
$$\Rightarrow DB^2 = (12)^2 + 5^2$$

$$\Rightarrow DB = \sqrt{144 + 25} = \sqrt{169}$$

$$\Rightarrow DB = 13 \text{ m.}$$

$$\text{Area of } \triangle DBC = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 12 \times 5 \text{ m}^2 = 30 \text{ m}^2$$



In $\triangle ABD$, $a = 9$ m, $b = 8$ m, $c = 13$ m

$$\therefore s = \frac{a+b+c}{2} = \frac{9+8+13}{2} \text{ m} = 15 \text{ m}$$

$$\begin{aligned} \therefore \text{Area of } \triangle ABD &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{15(15-9)(15-8)(15-13)} \text{ m}^2 \\ &= \sqrt{15 \times 6 \times 7 \times 2} \text{ m}^2 \\ &= \sqrt{1260} \text{ m}^2 = 35.5 \text{ m}^2 \text{ (approx.)} \end{aligned}$$

$$\begin{aligned} \therefore \text{Area of the park} &= \text{area of } \triangle DBC + \text{area of } \triangle ABD \\ &= (30 + 35.5) \text{ m}^2 = \mathbf{65.5 \text{ m}^2 \text{ Ans.}} \end{aligned}$$

Q.2. Find the area of a quadrilateral ABCD in which $AB = 3$ cm, $BC = 4$ cm, $CD = 4$ cm, $DA = 5$ cm and $AC = 5$ cm.

Sol. In $\triangle ABC$, we have

$$\begin{aligned} AB^2 + BC^2 &= 9 + 16 = 25 \\ &= AC^2 \end{aligned}$$

Hence, ABC is a right triangle, right angled at B
[By converse of Pythagoras theorem]

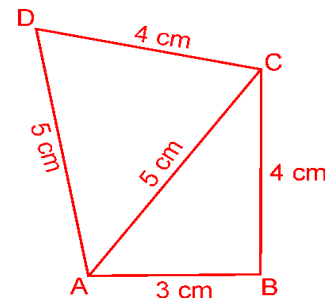
$$\begin{aligned} \therefore \text{Area of } \triangle ABC &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 3 \times 4 \text{ cm}^2 = 6 \text{ cm}^2. \end{aligned}$$

In $\triangle ACD$, $a = 5$ cm, $b = 4$ cm, $c = 5$ cm.

$$\therefore s = \frac{a+b+c}{2} = \frac{5+4+5}{2} \text{ cm} = 7 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of } \triangle ACD &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{7 \times (7-5)(7-4)(7-5)} \text{ cm}^2 = \sqrt{7 \times 2 \times 3 \times 2} \text{ cm}^2 \\ &= \sqrt{84} \text{ cm}^2 = 9.2 \text{ cm}^2 \text{ (approx.)} \end{aligned}$$

$$\begin{aligned} \therefore \text{Area of the quadrilateral} &= \text{area of } \triangle ABC + \text{area of } \triangle ACD \\ &= (6 + 9.2) \text{ cm}^2 = \mathbf{15.2 \text{ cm}^2 \text{ Ans.}} \end{aligned}$$



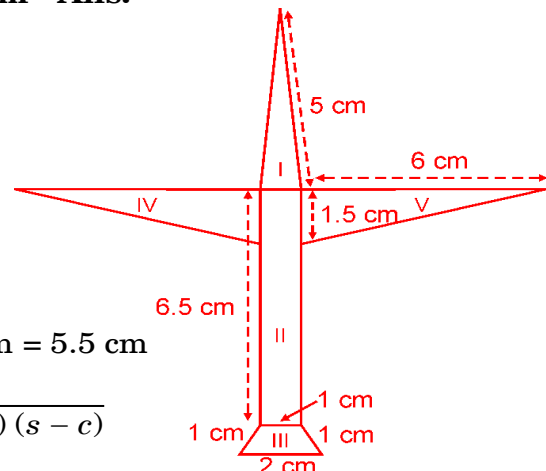
Q.3. Radha made a picture of an aeroplane with coloured paper as shown in the figure. Find the total area of the paper used.

Sol. For the triangle marked I :

$a = 5$ cm, $b = 5$ cm, $c = 1$ cm

$$\therefore s = \frac{a+b+c}{2} = \frac{5+5+1}{2} \text{ cm} = \frac{11}{2} \text{ cm} = 5.5 \text{ cm}$$

$$\text{Area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$



$$= \sqrt{5.5(5.5 - 5)(5.5 - 5)(5.5 - 1)} \text{ cm}^2$$

$$= \sqrt{5.5 \times 0.5 \times 0.5 \times 4.5} \text{ cm}^2 = \sqrt{6.1875} \text{ cm}^2 = 2.5 \text{ cm}^2$$

For the rectangle marked II :

Length = 6.5 cm, Breadth = 1 cm

Area of the rectangle = $6.5 \times 1 \text{ cm}^2 = 6.5 \text{ cm}^2$

For the trapezium marked III :

Draw $AF \parallel DC$ and $AE \perp BC$.

$AD = FC = 1 \text{ cm}$, $DC = AF = 1 \text{ cm}$

$\therefore BF = BC - FC = (2 - 1) \text{ cm} = 1 \text{ cm}$

Hence, $\triangle ABF$ is equilateral.

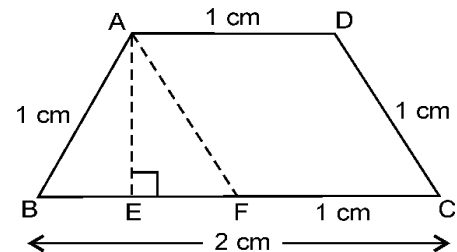
Also, E is the mid-point of BF.

$$\therefore BE = \frac{1}{2} \text{ cm} = 0.5 \text{ cm}$$

Also, $AB^2 = AE^2 + BE^2$ [Pythagoras theorem]

$$\Rightarrow AE^2 = 1^2 - (0.5)^2 = 0.75$$

$$\Rightarrow AE = 0.9 \text{ cm (approx.)}$$



Area of the trapezium = $\frac{1}{2}$ (sum of the parallel sides) \times distance between them.

$$= \frac{1}{2} \times (BC + AD) \times AE = \frac{1}{2} \times (2 + 1) \times 0.9 \text{ cm}^2 = 1.4 \text{ cm}^2.$$

For the triangle marked IV :

It is a right-triangle

$$\therefore \text{Area of the triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 6 \times 1.5 \text{ cm cm}^2 = 4.5 \text{ cm}^2.$$

For the triangle marked V :

This triangle is congruent to the triangle marked IV.

Hence, area of the triangle = 4.5 cm^2

Total area of the paper used = $(2.5 + 6.5 + 1.4 + 4.5 + 4.5) \text{ cm}^2$

$$= 19.4 \text{ cm}^2 \text{ Ans.}$$

Q.4. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

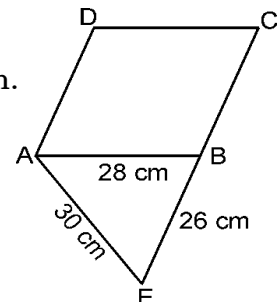
Sol. In the figure, ABCD is a parallelogram and ABE is the triangle which stands on the base AB

For the triangle ABE, $a = 30 \text{ cm}$, $b = 28 \text{ cm}$, $c = 26 \text{ cm}$.

$$\therefore s = \frac{a + b + c}{2} = \frac{30 + 28 + 26}{2} \text{ cm} = 42 \text{ cm}$$

$$\therefore \text{Area of the } \triangle ABE = \sqrt{s(s - a)(s - b)(s - c)}$$

$$= \sqrt{42(42 - 30)(42 - 28)(42 - 26)} \text{ cm}^2$$



$$= \sqrt{42 \times 12 \times 14 \times 16} \text{ cm}^2 = \sqrt{112896} \text{ cm}^2$$

$$= 336 \text{ cm}^2$$

Now, area of the parallelogram = base \times height

$$\Rightarrow 336 = 28 \times \text{height} \quad \begin{array}{l} \text{[Given, area of the triangle} \\ \text{= area of the parallelogram]} \end{array}$$

$$\Rightarrow \text{Height of the parallelogram} = \frac{336}{28} \text{ cm} = \mathbf{12 \text{ cm Ans.}}$$

Q.5. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

Sol. Clearly, the diagonal AC of the rhombus divides it into two congruent triangles.

For triangle ABC, $a = b = 30 \text{ m}$, $c = 48 \text{ m}$.

$$\therefore s = \frac{a + b + c}{2} = \frac{30 + 30 + 48}{2} \text{ m} = 54 \text{ m}$$

\therefore Area of the triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{54(54-30)(54-30)(54-48)} \text{ m}^2$$

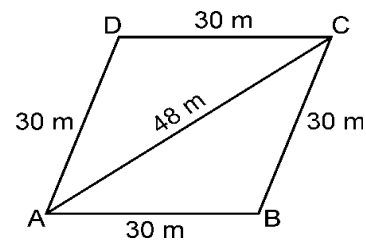
$$= \sqrt{54 \times 24 \times 24 \times 6} \text{ m}^2 = 432 \text{ m}^2$$

$$\therefore \text{Area of the rhombus} = 2 \times 432 \text{ m}^2 = 864 \text{ m}^2$$

Number of cows = 18

Hence, area of the grass field which each cow gets

$$= \frac{864}{18} \text{ m}^2 = \mathbf{48 \text{ m}^2 \text{ Ans.}}$$



Q.6. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see Fig.), each piece measuring 20 cm, 50 cm, and 50 cm. How much cloth of each colour is required for the umbrella?

Sol. First we find the area of one triangular piece.

Here, $a = b = 50 \text{ cm}$, $c = 20 \text{ cm}$

$$\therefore s = \frac{a + b + c}{2} = \frac{50 + 50 + 20}{2} \text{ cm} = 60 \text{ m}$$

$$\therefore \text{Area of one triangular piece} = \sqrt{s(s-a)(s-b)(s-c)}$$

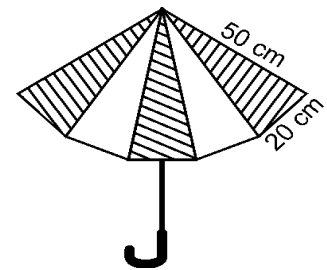
$$= \sqrt{60(60-50)(60-50)(60-20)} \text{ cm}^2$$

$$= \sqrt{60 \times 10 \times 10 \times 40} \text{ cm}^2 = 200\sqrt{6} \text{ cm}^2$$

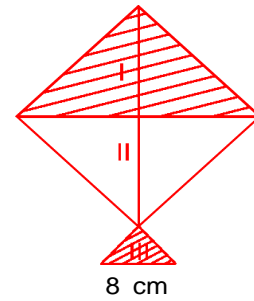
$$\therefore \text{Area of 10 such triangular pieces} = 10 \times 200\sqrt{6} \text{ cm}^2$$

$$= 2000\sqrt{6} \text{ cm}^2$$

$$\text{Hence, cloth required for each colour} = \frac{2000\sqrt{6}}{2} \text{ cm}^2 = \mathbf{1000\sqrt{6} \text{ cm}^2 \text{ Ans.}}$$



Q.7. A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in figure. How much paper of each shade has been used in it?



Sol. ABCD is a square.

So, $AO = OC = OB = OD$

and $\angle AOB = 90^\circ$ [Diagonals of a square bisect each other at right angles]

$BD = 32$ cm (Given) $\Rightarrow OA = \frac{32}{2}$ cm = 16 cm.

$\triangle ABD$ is a right triangle.

$$\begin{aligned} \text{So, area of } \triangle ABD &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 32 \times 16 \text{ cm}^2 = 256 \text{ cm}^2 \end{aligned}$$

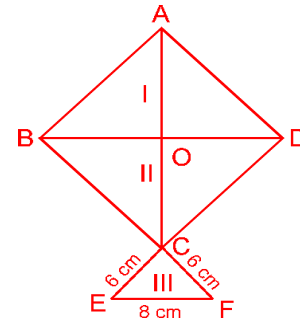
Thus, area of $\triangle BCD = 256 \text{ cm}^2$

For triangle CEF, $a = b = 6$ cm, $c = 8$ cm.

$$\therefore s = \frac{a + b + c}{2} = \frac{6 + 6 + 8}{2} \text{ cm} = 10 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of the triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{10(10-6)(10-6)(10-8)} \text{ cm}^2 \\ &= \sqrt{10 \times 4 \times 4 \times 2} \text{ cm}^2 = \sqrt{320} \text{ cm}^2 = 17.92 \text{ cm}^2 \end{aligned}$$

Hence, paper needed for shade I = **256 cm²**, for shade II = **256 cm²** and for shade III = **17.92 cm²** Ans.



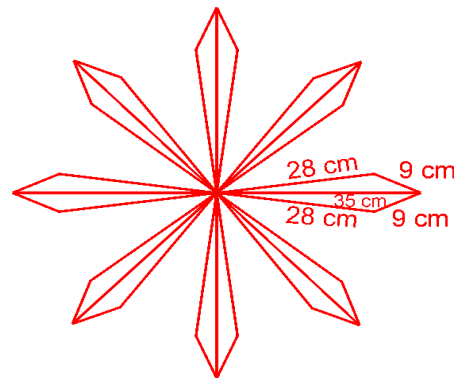
Q.8. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see figure). Find the cost of polishing the tiles at the rate of 50 p per cm².

Sol. We have lengths of the sides of 1 triangular tile are $a = 35$ cm, $b = 28$ cm, $c = 9$ cm.

$$\therefore s = \frac{a + b + c}{2} = \frac{35 + 28 + 9}{2} \text{ cm} = 36 \text{ cm}$$

$$\begin{aligned} \therefore \text{Area of 1 triangular tile} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{36(36-35)(36-28)(36-9)} \text{ cm}^2 \\ &= \sqrt{36 \times 1 \times 8 \times 27} \text{ cm}^2 = \sqrt{7776} \text{ cm}^2 = 88.2 \text{ cm}^2 \end{aligned}$$

\therefore Area of 16 such tiles = $16 \times 88.2 \text{ cm}^2$



Cost of polishing $1 \text{ cm}^2 = 50 \text{ p} = \text{Re } 0.50$

$$\begin{aligned} \therefore \text{Total cost of polishing the floral design} &= \text{Rs } 16 \times 88.2 \times 0.50 \\ &= \text{Rs } 705.60 \text{ Ans.} \end{aligned}$$

Q.9. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

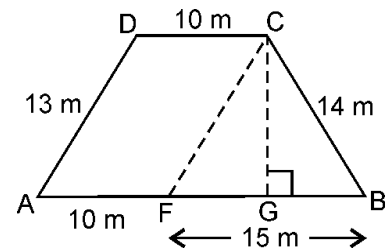
Sol. In the figure ABCD is the field. Draw $CF \parallel DA$ and $CG \perp AB$.

$$DC = AF = 10 \text{ m, } AD = FC = 13 \text{ m}$$

$$\text{For } \triangle BCF, a = 15 \text{ m, } b = 14 \text{ m, } c = 13 \text{ m}$$

$$\therefore s = \frac{a + b + c}{2} = \frac{15 + 14 + 13}{2} \text{ m} = 21 \text{ m}$$

$$\begin{aligned} \therefore \text{Area of } \triangle BCF &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{21(21-15)(21-14)(21-13)} \text{ m}^2 \\ &= \sqrt{21 \times 6 \times 7 \times 8} \text{ m}^2 \\ &= \sqrt{7056} \text{ cm}^2 = 84 \text{ m}^2 \end{aligned}$$



$$\text{Also, area of } \triangle BCF = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times BF \times CG$$

$$\Rightarrow 84 = \frac{1}{2} \times 15 \times CG$$

$$\Rightarrow CG = \frac{84 \times 2}{15} \text{ m} = 11.2 \text{ m}$$

\therefore Area of the trapezium = $\frac{1}{2} \times$ sum of the parallel sides \times distance between them.

$$= \frac{1}{2} \times (25 + 10) \times 11.2 \text{ m}^2$$

$$= 196 \text{ m}^2$$

Hence, area of the field = **196 m² Ans.**